*Chapter 12*

# THERMODYNAMIC ANALYSES OF POWER PLANTS

It sounded an excellent plan, no doubt, and very neatly and simply arranged; the only difficulty was that she had not the smallest idea how to set about it.

– Lewis Carrol (*Alice in Wonderland*)

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Power plants are where power is produced such as in the electricity generating stations and turbojet engines. The workings of these power plants are complicated. In this chapter, however, we will learn about the basic working principles governing a few simple power plants and about carrying out thermodynamic analyses of simplified power plants to determine vital parameters such as the overall thermal efficiency. This chapter is written to make the students of this book appreciate some real life applications of what they have so far learned in thermodynamics.

# 12.1 Gas Turbine for Electric Power Generation

Water, as we have seen in Chapter 1, is used to rotate the shaft of a turbine in a hydroelectric power station. In a gas turbine used for electric power generation, the shaft of the turbine is rotated by the gases produced in the burning of a fuel with air. The basic working principle of a simple gas turbine used for electric power generation is explained below with the help of the flow diagram shown in Figure 12.1.

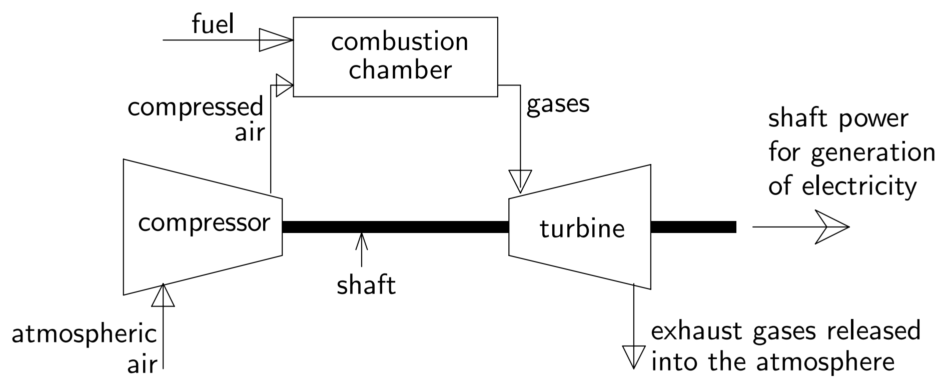


Figure 12.1 Flow diagram of a simple gas turbine power plant.

Air is drawn from the atmosphere and is compressed to a high pressure in a compressor. The high pressure air enters a combustion chamber, in which fuel is sprayed onto the compressed air, and the fuel-air mixture is burned at constant pressure. The gases leaving the combustion chamber at high pressure and high temperature are directed towards the turbine blades so as to rotate the turbine shaft. In general, nearly half of the work output of the rotating turbine shaft is used to rotate the compressor shaft, and the rest is used to produce electricity by spinning a coil between the poles of a magnet or an electromagnet. Thus, part of the heat generated in burning the fuel in the combustion chamber is converted into useful electrical energy.

The gases leaving the turbine are released into the atmosphere. These gases contain carbon dioxide, nitrous oxides, sulfur oxides and particulate matter. The temperature of the exhaust gases can also be very high. Before releasing these gases into the atmosphere, therefore, it is essential to make sure that the polluting potential of these exhaust gases is reduced to the level set by the environmental authority of the country concerned. Even though there is absolutely nothing one could do about the carbon dioxide, a product of complete combustion of the fuel used, into the atmosphere.

**Example 12.1**

Consider the gas turbine power plant shown in Figure 12.1. Atmospheric air at 1 bar and 300 K is drawn into a compressor at a mass flow rate of 350 kg/s, and is compressed to 5 bar. The compressed air is heated at constant pressure in the combustion chamber by burning the fuel injected onto the air flowing through the combustion chamber. The gases leave the combustion chamber at 1200 K, and enter the turbine of the power plant, rotate the turbine shaft, and leave the turbine at 1 bar.

(a) Determine the power input to the compressor, power production of the turbine, and the power available for electricity generation. Also, determine the back work ratio, defined as the ratio of the compressor work to the turbine work.

(b) Calculate the heat input to the combustion chamber. Also, determine the thermal efficiency of the power plant, defined as the ratio of the net work output to the heat input.

(c) Determine the air-fuel ratio, assuming the heating value of the fuel as 42 MJ/kg.

(d) Discuss the validity of the solutions obtained above.

### Solution to Example 12.1

The block diagram of the given problem is shown in Figure 12.2.

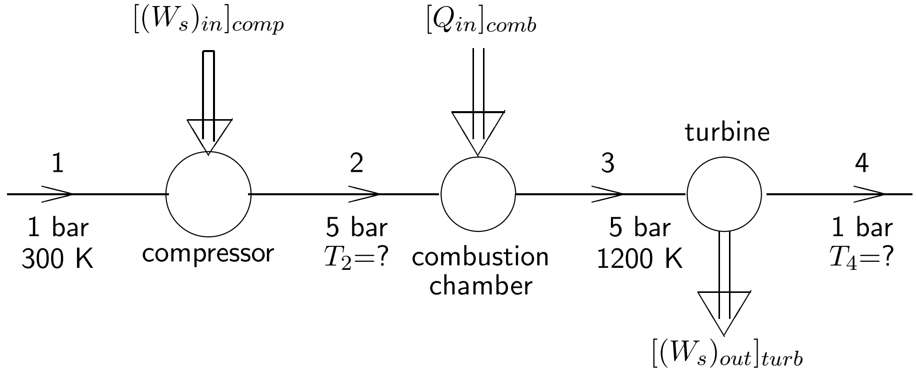


Figure 12.2 Block diagram for Example 12.1.

Let us work out the problem under ideal conditions making the following assumptions:

Assumption 1: Assume that the flows through the compressor and the turbine are adiabatic and reversible. Then, (7.31) could be used to relate the pressure and the temperature at the inlet to the pressure and the temperature at the exit.

Assumption 2: Neglect the increase in the mass flow rate of gases leaving the combustion chamber as a result of fuel injection. That is, the mass flow rates through the compressor, combustion chamber and the turbine are all 350 kg/s.

Assumption 3: Assume $C_p$ as 1.005 kJ/kg·K and $\gamma$ as 1.4 for both the air and the combustion gases.

Assumption 4: Assume that all the gases involved in the problem behave like ideal gases, with constant specific heats.

Assumption 5: Neglect the potential and kinetic energy changes.

(a) First, let us determine the power input to the compressor. Using (10.11), we have

$$[(\dot{W}_s)_{in}]_{comp} = \dot{m}\, C_p\, (T_2 - T_1) = 350 \times 1.005 \times (T_2 - 300) \ \text{kJ/s}$$

of which $T_2$ is unknown. Since the flow through the compressor is assumed to be reversible and adiabatic, (7.31) gives

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 300 \text{ K} \times \left(\frac{5}{1}\right)^{0.4/1.4} = 475.2 \text{ K}$$

Therefore, $[(\dot{W}_s)_{in}]_{comp}$ = 61.63 MJ/s. That is, the power requirement of the compressor is 61.63 MW.

Let us now determine the work output of the turbine. Using (10.9), we have

$$[(\dot{W}_s)_{out}]_{turb} = \dot{m}\, C_p\, (T_3 - T_4) = 350 \times 1.005 \times (1200 - T_4) \text{ kJ/s}$$

of which $T_4$ is unknown. Since the flow through the turbine is assumed to be reversible and adiabatic, (7.31) gives

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} = 1200 \text{ K} \times \left(\frac{1}{5}\right)^{0.4/1.4} = 757.7 \text{ K}$$

Therefore, $[(\dot{W}_s)_{out}]_{turb}$ = 155.58 MJ/s. That is, the power output of the turbine is 155.58 MW.

The power available for electricity generation is the difference between the power output of the turbine (= 155.58 MW) and the power requirement of the compressor (= 61.63 MW), which is 93.95 MW.

The back work ratio is calculated as

$$\text{bwr} = \frac{\text{compressor work input}}{\text{turbine work output}} = \frac{61.63 \text{ MW}}{155.58 \text{ MW}} = 39.6\%$$

**Comment:** About 39.6% of the work produced by the turbine is consumed by the compressor to compress the ambient air to 5 bar pressure. Only the remaining 60.4% is available for electricity generation. One of the major drawbacks in using a gas turbine power plant for electricity generation is the relatively large part of the turbine work output being consumed by the compressor.

(b) Heat input to the combustion chamber can be calculated by applying (10.3) to the air flowing through the combustion chamber as

$$\begin{aligned} [\dot{Q}_{in}]_{comb} &= \dot{m}\, C_p\, (T_3 - T_2) \\ &= 350 \times 1.005 \times (1200 - 475.2) \text{ kJ/s} = 254.95 \text{ MW} \end{aligned}$$

where the fuel flow rate through the combustion chamber is neglected.

The thermal efficiency of the gas turbine plant is calculated as

$$\begin{aligned} \eta_{th} &= \frac{\text{net work output of the gas turbine plant}}{\text{heat input to the gas turbine plant}} \\ &= \frac{155.58 \text{ MW} - 61.63 \text{ MW}}{254.95 \text{ MW}} \\ &= \frac{93.95}{254.95} = 36.9\% \end{aligned}$$

**Comment:** Only 36.9% of the 254.95 MW heat input to the combustion chamber is available for electricity generation. The remaining 63.1%, which is about 161 MW, ends up in the atmosphere with the exhaust gases released into the atmosphere at 757.7 K by the turbine. Note that for each MW of electric power generation there is at least 1.7 (=161/93.95) MW of power wasted. All this waste energy reaching the environment is a source of thermal pollution.

(c) Since the heating value of the fuel is assumed to be 42 MJ/kg, the mass flow rate of fuel supplied to the combustion chamber can be calculated as

$$\begin{aligned} \text{mass flow rate of the fuel} &= \frac{[\dot{Q}_{in}]_{comb}}{\text{heating value of the fuel}} \\ &= \frac{254.95 \text{ MJ/s}}{42 \text{ MJ/kg}} \\ &= 6.07 \text{ kg/s} \end{aligned}$$

**Comment:** The fuel-air mass ratio is therefore about 0.017. In other words, the fuel mass flow rate is only about 1.7% of the air mass flow rate.

(d) The solutions obtained above are valid only under the five assumptions made. In reality, Assumption 1 hardly holds. That is, the flow through the compressor and the flow through the turbine are far from reversible and adiabatic in reality. Therefore, the actual exit temperatures of the air leaving the compressor and the gases leaving the turbine would be very different from those calculated in part (a). It will cause the actual amount of work input to the compressor to increase and the actual amount of work produced by the turbine to decrease. These differences will cause the back work ratio to increase and the thermal efficiency to decrease.

Assumption 2 is also not valid since the mass flow rate through the turbine is greater than the mass flow rate through the compressor, and the difference is the mass flow of the fuel added to the combustion chamber. However, since the fuel mass flow rate is only about 1.7% of the air mass flow rate, as calculated in part (c), Assumption 2 would not have introduced any appreciable error. For the same reason, treating the combustion gases as air, as per Assumption 3, would not have introduced any appreciable error, either. Assumptions 4 and 5, however, would introduce some bias in the solutions obtained.

Nevertheless, the solutions obtained above under ideal conditions certainly give a very good idea about the performance of the gas turbine. For example, the efficiency of the gas turbine in reality will always be less than, and never more than, 36.9%. And, more than 63.1% of the heat input to the combustion chamber will be lost to the environment, causing thermal pollution.

## Example 12.2

Rework **Example 12.1** with a regenerator unit included in the gas turbine power plant as shown in Figure 12.3.

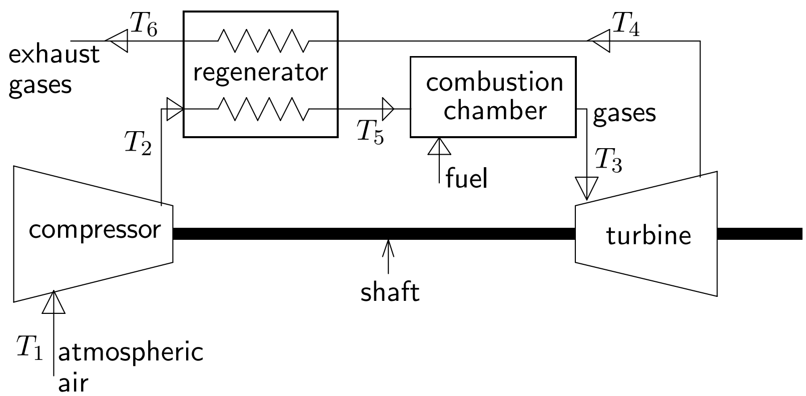


Figure 12.3 Flow diagram of a gas turbine power plant with regeneration.

A regenerator is used to recover part of the energy lost to the environment with the exhaust gases leaving the turbine at high temperatures. It is simply a heat exchanger in which the compressed air is preheated by the hot gases leaving the turbine as shown in the figure. Let us assume that adiabatic conditions prevails at the regenerator and that there is no pressure drop across it.

(a) Determine the power input to the compressor, power production of the turbine, and the power available for electricity generation. Determine also the back work ratio.

(b) Calculate the heat input to the combustion chamber, and the thermal efficiency of the gas turbine plant.

### Solution to Example 12.2

(a) The pressure and the temperature of the air entering the compressor, the pressure of the air leaving the compressor, the temperature of the gases entering the turbine, and the pressure of the gases leaving the turbine are all the same as in **Example 12.1**. Hence, the exit temperature of the air leaving the compressor, the work input to compressor, the exit temperature of the gases leaving the turbine and the work produced by the turbine are all the same as those calculated in the **Solution to Example 12.1**.

That is, $T_2$ = 475.2 K, $[(\dot{W}_s)_{in}]_{comp}$ = 61.63 MW, $T_4$= 757.7 K, and $[(\dot{W}_s)_{out}]_{turb}$ = 155.58 MW. Therefore, the power available for electricity generation and the back work ratio are the same as those calculated in the **Solution to Example 12.1**, which are 93.95 MW and 36.9%, respectively.

(b) The use of regenerator would cause the heat input to the combustion chamber to be lower than that was calculated in the **Solution to Example 12.1**, since the compressed air is preheated by the hot gases leaving the turbine in the regenerator before it enters the combustion chamber.

Since the regenerator is a heat exchanger, assuming adiabatic condition, (10.19) can be used to get

$$\frac{\dot{m}_{gases}}{\dot{m}_{air}} = \frac{(C_p)_{air}(T_5 - T_2)}{(C_p)_{gases}(T_4 - T_6)}$$

Since the mass flow rate of the gases is assumed to be the same as that of the air, $C_p$ is assumed to be the same for both air and gases, $T_2$ = 475.2 K and $T_4$ = 757.7 K, we get

$$T_5 - 475.2 \text{ K} = 757.7 \text{ K} - T_6$$

where $T_5$ and $T_6$ are unknown.

For the heat to flow from the gases to the air within the heat exchanger, $T_6$ should be higher than $T_2$ (= 475.2 K). If, for example, we take $T_6$ to be 500 K then $T_5$ would be 732.9 K. The heat input to the air flowing through the combustion chamber can then be determined as

$$\begin{aligned} [\dot{Q}_{in}]_{comb} &= \dot{m}\, C_p\, (T_3 - T_5) \\ &= 350 \times 1.005 \times (1200 - 732.9) \text{ kJ/s} = 164.30 \text{ MJ/s} \end{aligned}$$

which is about 90.65 MW less than the heat input to the combustion chamber without a regenerator as in the **Example 12.1**.

The thermal efficiency of the gas turbine plant is calculated as

$$\begin{aligned} \eta_{th} &= \frac{\text{net work output of the gas turbine plant}}{\text{heat input to the gas turbine plant}} \\ &= \frac{155.58 \text{ MW} - 61.63 \text{ MW}}{164.30 \text{ MW}} = \frac{93.95}{164.30} = 57.2\% \end{aligned}$$

**Comment:** When using a regenerator to preheat the compressed air by the hot gases leaving the turbine, the heat requirement of the combustion chamber is reduced. So that the thermal efficiency of the gas turbine power plant increases to 57.2% from 36.9%, obtained without the regenerator in the **Solution to Example 12.1**. That is, about 57.2% of the 164.30 MW heat input to the combustion chamber is available for electricity generation. The remaining 42.8% of the heat input, which is about 70 MW, reaches the atmosphere with the exhaust gases released into the atmosphere at 500 K by the regenerator. Note that for each MW of electric power generation there is about 0.75 (=70/93.95) MW of power wasted into the environment. Thermal pollution caused by the gas turbine power plant with a regenerator is therefore considerably lower than the thermal pollution caused by the gas turbine power plant without a regenerator.

## 12.2 Gas Turbine for Jet Propulsion

Gas turbine is an essential part of a turbojet engine commonly used for aircraft propulsion. A turbojet engine consists of a diffuser, compressor, combustion chamber, turbine and a nozzle, as shown in the schematic of a turbojet engine in Figure 12.4.

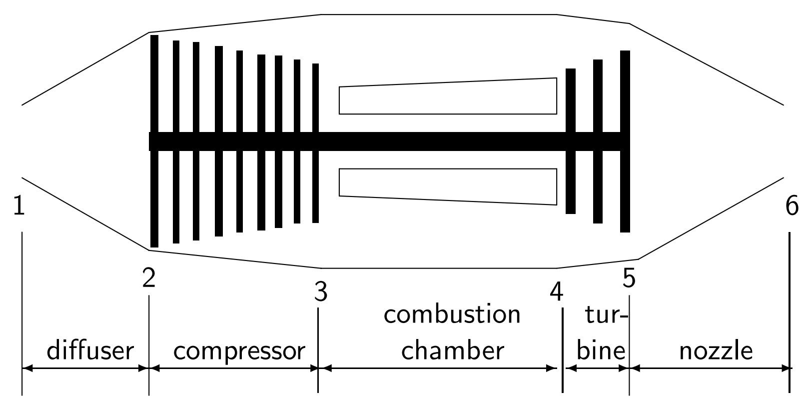


Figure 12.4 Schematic of a turbojet engine.

The inlet of a turbojet engine is shaped into a diffuser in which the ambient air entering the turbojet engine is slowed down to near zero speed. Reduction in the speed of the air flowing through the diffuser is accompanied by increase in the air pressure. The slightly compressed air enters the compressor in which it is compressed to a high pressure. The exit pressure of the compressor can be 7 to 25 times higher than the inlet pressure. The high pressure air leaving the compressor enters the combustion chamber, where fuel is sprayed onto the compressed air and the resultant mixture is burned. The hot products of combustion at a high pressure and a temperature enter the turbine, and sets the turbine shaft on rotation.

The compressor-combustion chamber-turbine combination of the turbojet engine functions in a manner very similar to that of the gas turbine power plant discussed in Section 12.1. In the turbojet engine, however, turbine produces just enough power to drive the compressor shaft and some other devices such as a small generator. Therefore, almost all the work output of the turbine can be considered as being consumed by the compressor in the turbojet engine, unlike in the case of the gas turbine power plant in which a good portion of the turbine power output is used for the generation of electricity.

The gases leaving the turbine of the turbojet engine enter a nozzle, and attain high speed as they flow through the nozzle which is accompanied by a pressure reduction. The high speed gases leaving the turbojet engine imparts a thrust on the aircraft so as to propel the aircraft forward. The thrust equation for a turbojet engine can be derived from the Newton's second law, for the case where the inlet pressure is the same as the exit pressure, as

$$\text{Net Thrust} = \dot{m}_{air} \left(\mathcal{V}_{exit} - \mathcal{V}_{inlet}\right) \tag{12.1}$$

where $\mathcal{V}_{exit}$ is the exit velocity of the gases leaving the nozzle relative to the aircraft and $\mathcal{V}_{inlet}$ is the inlet velocity of the air entering the diffuser relative to the aircraft. In Figure 12.4, the inlet is marked by 1 and the exit is marked by 6. In deriving (12.1), we have omitted the fact that the mass flow rate of the gases leaving the turbojet engine differs from the mass flow rate of air entering the engine by the mass flow rate of the fuel supplied to the combustion chamber. Since the fuel-air mass ratio used in the turbojet engine is usually very small, the above omission does not introduce any appreciable error in the calculation of the net thrust.

## Example 12.3

Air enters a diffuser of the turbojet engine of an aircraft flying at a speed of 300 m/s at an altitude where the pressure is 0.95 bar and the temperature is $10^o$C. Air leaving the diffuser with a negligibly small speed enters the compressor which has a pressure ratio of 9:1. Each kg of compressed air entering the combustion chamber receives about 650 kJ of heat as it passes through the combustion chamber at constant pressure. The gases leaving the combustion chamber enter the turbine at a high temperature and a pressure, set the turbine shaft on rotation, and leave the turbine. The gases leaving the turbine pass through the nozzle where they achieve high speeds. The turbojet engine is so designed that the pressure of the gases at the exit of the nozzle falls back to 0.95 bar. Assume that the flow within the turbojet engine except in the combustion chamber is reversible and adiabatic and that all the work output of the turbine is used to drive the compressor. Neglect the speed of the flow through the turbojet engine except at the inlet and the exit of the engine. Also, neglect the effect of fuel flow rate.

(a) Determine the pressures and the temperatures at the inlets and the outlets of the diffuser, compressor, combustion chamber, turbine and the nozzle of the turbojet engine.

(b) Determine the speed of the gases at the nozzle exit.

(c) Compute the forward thrust imparted on the turbojet engine per kg of the air flowing through the engine.

### Solution to Example 12.3

Let us use the schematic of the turbojet engine shown in Figure 12.4 and the labels 1, 2, 3, 4, 5 and 6 on it to mark the states of the fluid entering/leaving the diffuser, compressor, combustion chamber, turbine and the nozzle of the turbojet engine.

Flow through the Diffuser:

At the diffuser inlet, $P_1 = 0.95$ bar, $T_1 = 283$ K and $c_1 = 300$ m/s, where $c_1$ denotes the speed of the air at the diffuser inlet. At the diffuser exit, $c_2 = 0$, where $c_2$ denotes the speed of the air at the diffuser exit.

The steady flow energy equation applicable for the adiabatic flow through a diffuser is used to determine $T_2$ as

$$T_2 = T_1 + \frac{c_1^2 - c_2^2}{2 \times C_p} = 283 \text{ K} + \frac{300^2 \text{ m}^2/\text{s}^2}{2 \times 1005 \text{ J/kg} \cdot \text{K}} = 327.8 \text{ K}$$

Since the flow through the diffuser is assumed to be reversible and adiabatic, (7.31) gives

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = 0.95 \text{ bar} \times \left(\frac{327.8}{283}\right)^{1.4/0.4} = 1.59 \text{ bar}$$

$P_2 = 1.59$ bar and $T_2 = 327.8$ K at the diffuser exit, which is also the compressor inlet. The speed of the flow is assumed to be negligible at this cross-section.

Flow through the Compressor:

The compressor has a pressure ratio of 9:1, so that

$$P_3 = 9 \times P_2 = 9 \times 1.59 \text{ bar} = 14.31 \text{ bar}$$

Since the flow through the compressor is assumed to be reversible and adiabatic, (7.31) gives

$$T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{(\gamma-1)/\gamma} = 327.8 \text{ K} \times \left(\frac{9 \times P_2}{P_2}\right)^{0.4/1.4} = 614.1 \text{ K}$$

$P_3 = 14.31$ bar and $T_3 = 614.1$ K at the compressor exit, which is also the combustion chamber inlet. The speed of the flow is assumed to be negligible at this cross-section.

Flow through the Combustion Chamber:

The flow through the combustion chamber is assumed to be at constant pressure. So that

$$P_4 = P_3 = 14.31 \text{ bar}$$

Each kg of compressed air entering the combustion chamber receives about 650 kJ of heat as it passes through the combustion chamber. Neglecting the effect of fuel flow rate, applying the steady flow energy equation to the flow through the combustion chamber gives

$$650 \text{ kJ/kg} = C_p \,(T_4 - T_3)$$

from which $T_4$ can be found as

$$T_4 = T_3 + \frac{650 \text{ kJ/kg}}{C_p} = \left(614.1 + \frac{650}{1.005}\right) \text{ K} = 1260.9 \text{ K}$$

$P_4 = 14.31$ bar and $T_4 = 1260.9$ K at the combustion chamber exit, which is also the turbine inlet. The speed of the flow is assumed to be negligible at this cross-section.

Flow through the Turbine:

Since all the work output of the turbine is assumed to be used to drive the the compressor, we have

$$[(\dot{W}_s)_{out}]_{turbine} = [(\dot{W}_s)_{in}]_{compressor}$$

Using the steady flow energy equation to both the turbine and the compressor, we can expand the above equation to

$$\dot{m}_{gases}\,(C_p)_{gases}\,(T_4 - T_5) = \dot{m}_{air}\,(C_p)_{air}\,(T_3 - T_2)$$

Since the effect of fuel flow rate is ignored, the mass flow rates of the gases and of air can be taken as equal. The values of $C_p$ for air and for the gases can be taken as equal as well. Therefore, we can calculate $T_5$ using

$$T_5 = T_4 - T_3 + T_2 = (1260.9 - 614.1 + 327.8)\ \text{K} = 974.6\ \text{K}$$

Since the flow through the turbine is assumed to be reversible and adiabatic, (7.31) gives

$$P_5 = P_4 \left(\frac{T_5}{T_4}\right)^{\gamma/(\gamma-1)} = 14.31\ \text{bar} \times \left(\frac{974.6}{1260.9}\right)^{1.4/0.4} = 5.81\ \text{bar}$$

$P_5 = 5.81$ bar and $T_5 = 974.6$ K at the turbine exit, which is also the nozzle inlet. The speed of the flow is assumed to be negligible at this cross-section.

Flow through the Nozzle:

The pressure of the gases at the exit of the nozzle falls back to 0.95 bar. Therefore

$$P_6 = 0.95\ \text{bar}$$

Since the flow through the nozzle is assumed to be reversible and adiabatic, (7.31) gives

$$T_6 = T_5 \left(\frac{P_6}{P_5}\right)^{(\gamma-1)/\gamma} = 974.6\ \text{K} \times \left(\frac{0.95}{5.81}\right)^{0.4/1.4} = 580.9\ \text{K}$$

Steady flow energy equation is applied to the flow through the nozzle to determine $c_6$ as

$$\begin{aligned} c_6 &= \sqrt{2 \times C_p\,(T_5 - T_6)} \\ &= \sqrt{2 \times 1005 \times (974.6 - 580.9)\ \text{J/kg}} = 890\ \text{m/s} \end{aligned}$$

$P_6 = 0.95$ bar and $T_6 = 580.9$ K at the nozzle exit. The speed of the flow at the nozzle exit is 890 m/s.

**Comment:** Such high speed at the nozzle exit is achieved under the assumed ideal conditions, such as the flow within the turbojet engine except in the combustion chamber being reversible and adiabatic.

Forward thrust imparted on the turbojet engine:

Since the exit pressure is the same as the inlet pressure, the forward thrust imparted on the turbojet engine per kg of the air flowing through the engine is calculated using (12.1) as

$$\frac{\text{Net Thrust}}{\dot{m}_{air}} = (c_6 - c_1) = (890 - 300)\ \text{m/s} = 590\ \text{N per kg/s}$$

## 12.3 Steam Turbine for Electric Power Generation

In a gas turbine used for electric power generation, the shaft of the turbine is rotated by the gases produced in the burning of a fuel with air. In a steam turbine used for electric power generation, superheated steam is used to rotate the shaft of the turbine. The basic working principle of a simple steam turbine used for electric power generation is explained below with the help of the flow diagram shown in Figure 12.5.

Saturated water entering the pump is compressed to a high pressure and the compressed water is fed to the steam generator, which is sometimes referred to as the boiler. The compressed water is heated to superheated steam state in the steam generator, which is a large heat exchanger where the heat is transferred from the hot combustion gases to the water. The superheated steam leaving the steam generator at a high pressure and a temperature enters the turbine, where it expands rotating the turbine shaft.

The work output of the rotating turbine shaft is used to produce electricity by spinning a coil between the poles of a magnet or an electromagnet. Thus, part of the heat transferred from the hot combustion gases to the water in the steam generator is converted into useful electrical energy. The wet steam leaving the turbine is condensed to saturated water state in a condenser, which is also a large heat exchanger where the heat is transferred from the steam to cooling water. The saturated water leaving the condenser enters the pump, thus making a cyclic flow through the pump, steam generator, turbine and condenser.

The combustion gases leaving the steam generator, rich in pollutants such as the greenhouse gas carbon dioxide, enter the atmosphere. The cooling water leaving the condenser at an elevated temperature is cooled in the cooling towers, by transferring considerable amount of heat to the atmosphere, and thereby causing thermal pollution.

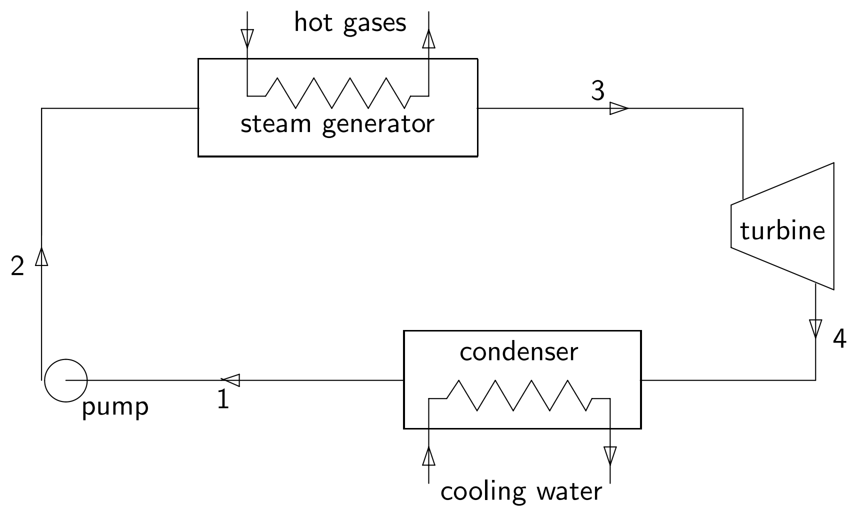


Figure 12.5 Flow diagram of a simple steam turbine power plant.

**Example 12.4**

In the steam turbine power plant, shown in Figure 12.5, assume that saturated water at 0.08 bar flowing at a rate of 50 kg/s is compressed by the pump to 70 bar. The compressed water is fed to the steam generator where it is converted to superheated steam at 450°C at the same pressure. The superheated steam expanding through the turbine, leaves the turbine as wet steam at 0.08 bar. The wet steam is condensed to saturated water state in the condenser, thus completing the cycle.

(a) Determine the enthalpies of the flow at the inlets and the outlets of the turbine and the pump, assuming reversible adiabatic flows through the turbine and the pump.
(b) Determine the power production at the turbine, power requirement of the pump, and the back work ratio, defined as the ratio of the pump work to the turbine work.
(c) Determine the heat input at the steam generator, and the thermal efficiency of the steam turbine plant, defined as the ratio of net work out to heat in.
(d) Determine the heat rejected at the condenser.
(e) Calculate the cooling water mass flow rate if the cooling water temperature is raised from 32°C to about 2.5°C less than the temperature of the wet steam condensing in the condenser. Take cooling water $C_p$ as 4.2 kJ/kg · K.

## Solution to Example 12.4

(a) Let us use the flow diagram of the simple steam turbine power plant shown in Figure 12.5 and the labels 1, 2, 3 and 4 on it to mark the states of water/steam entering or leaving the components of the steam turbine power plant.

At state 3, which is the turbine inlet, we have superheated steam at 70 bar and 450°C. The specific enthalpy found from a Superheated Steam Table is $h_3 = 3287$ kJ/kg.

At state 4, which is the turbine outlet, we have wet steam at 0.08 bar. Since the flow through the turbine is assumed to be reversible adiabatic, $s_4 = s_3 = s$ at 70 bar and 450°C $= 6.632$ kJ/kg · K.

For state 4 at 0.08 bar and $s_4 = 6.632$ kJ/kg · K, the dryness fraction can be calculated using

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.632 - 0.593}{7.634} = 0.7911$$

The specific enthalpy is then

$$h_4 = h_f + x_4 \times h_{fg} = 174 + 0.7911 \times 2402 = 2074 \text{ kJ/kg}$$

At state 1, which is the pump inlet, we have saturated water at 0.08 bar. The specific enthalpy is $h_1 = 174$ kJ/kg.

At state 2, which is the pump outlet, we have compressed water at 70 bar. Since the flow through the pump is assumed to be reversible adiabatic, $s_2 = s_1 = s_f$ at 0.08 bar $= 0.593$ kJ/kg·K.

State 2 at 70 bar and $s_2 = 0.593$ kJ/kg·K is a compressed water state since $s_f$ at 70 bar is 3.122 kJ/kg·K. We should therefore be able to find $h_2$ using a Compressed Water Table. However, since data for compressed water is not easily found, let us use the following convenient but an approximate method to find $h_2$.

The work input to a pump with adiabatic flow is expressed by (10.11). The work input to a pump with reversible flow is approximated by (11.28). Since the given flow is reversible adiabatic, we combine (10.11) and (11.28) to get $h_2$ as follows:

$$\begin{aligned} \dot{m}(h_2 - h_1) &\approx \dot{m}\, v_1 \,(P_2 - P_1) \\ h_2 &\approx h_1 + v_1 \,(P_2 - P_1) \end{aligned} \tag{12.2}$$

where $h_1$ is known, $v_1 = v_f$ at 0.08 bar $= 0.0010084$ m$^3$/kg, $P_1 = 0.08$ bar and $P_2 = 70$ bar. Therefore, we have

$$h_2 \approx 174 \text{ kJ/kg} + 0.0010084 \text{ m}^3\text{/kg} \times (7000 - 8) \text{ kPa} = 181 \text{ kJ/kg}$$

Summarizing the results obtained above, we have

| State | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Condition | saturated water | compressed water | superheated steam | wet steam |
| $P$ (in bar) | 0.08 | 70 | 70 | 0.08 |
| $T$ (in °C) | 41.5 | - | 450 | 41.5 |
| $h$ (in kJ/kg) | 174 | 181 | 3287 | 2074 |
| $x$ | 0 | - | - | 79.11% |

(b) The power production of the turbine must be determined. Using (10.9) for the flow through an adiabatic turbine, we have

$$[(\dot{W}_s)_{out}]_{turb} = \dot{m}(h_3 - h_4) = 50 \times (3287 - 2074) \text{ kJ/s} = 60.65 \text{ MW}$$

The power requirement of the pump must be determined. Using (10.11) for the flow through an adiabatic pump, we have

$$[(\dot{W}_s)_{in}]_{pump} = \dot{m}(h_2 - h_1) = 50 \times (181 - 174) = 0.35 \text{ MW}$$

The back work ratio is calculated as

$$\text{bwr} = \frac{\text{pump work input}}{\text{turbine work output}} = \frac{0.35 \text{ MW}}{60.65 \text{ MW}} = 0.6\%$$

**Comment:** The power consumption of the pump to compress water from 0.08 bar to 70 bar is only about 0.6% of the power production of the turbine. That is, almost all the work produced by the turbine is available for electricity generation. It is an advantage that a steam turbine power plant has over a gas turbine power plant.

(c) Heat input to the steam generator is calculated by applying (10.3) to the fluid flowing through the steam generator as

$$\begin{aligned}[\dot{Q}_{in}]_{steam\ generator} &= \dot{m}\,(h_3 - h_2) \\ &= 50 \times (3287 - 181)\ \text{kJ/s} = 155.30\ \text{MW}\end{aligned}$$

The thermal efficiency of the steam turbine plant is calculated as

$$\begin{aligned}\eta_{th} &= \frac{\text{net work out}}{\text{heat in}} \\ &= \frac{60.65\ \text{MW} - 0.35\ \text{MW}}{155.30\ \text{MW}} = \frac{60.30}{155.30} = 38.8\%\end{aligned}$$

**Comment:** Only about 38.8% of the 155.30 MW heat input to the steam generator is available for electricity generation.

(d) Heat rejected at the condenser is calculated by applying (10.3) to the fluid flowing through the condenser as

$$\begin{aligned}[\dot{Q}_{out}]_{condenser} &= \dot{m}\,(h_4 - h_1) \\ &= 50 \times (2074 - 174)\ \text{kJ/s} = 95.00\ \text{MW}\end{aligned}$$

**Comment:** The percentage of the heat input in the steam generator that is rejected by the condenser is calculated using (95.00/155.30)×100, which equals 61.2%. In other words, for each MW of electric power generation there is at least about 1.6 (=95.00/60.30) MW of power is wasted. All this waste energy reaching the environment is a source of thermal pollution.

(e) Steam condenses in the condenser at the saturated temperature at 0.08 bar, which is 41.5°C. The cooling water temperature is therefore raised from 32°C to 39°C. Assume that the cooling water absorbs all that 95.00 MW of heat rejected at the condenser. The mass flow rate of the cooling water is calculated as

$$\begin{aligned}\dot{m}_{cooling\ water} &= \frac{95.00\ \text{MW}}{4.2 \times (39 - 32)\ \text{kJ/kg}} \\ &= 3231\ \text{kg/s} = 11.63 \times 10^6\ \text{kg/h}\end{aligned}$$

**Example 12.5**

Consider the schematic of the steam turbine power plant shown in Figure 12.6. Saturated water at 0.08 bar flowing at a rate of 50 kg/s is compressed reversibly and adiabatically by a pump to 70 bar. The compressed water is fed to a steam generator where it is converted to superheated steam at 450°C at the same pressure. The steam flows reversibly and adiabatically through a HP (high pressure) turbine, and leaves the turbine at 7 bar. The steam leaving the turbine is reheated to 425°C, and fed to the LP (low pressure) turbine. The steam flowing reversibly and adiabatically through the LP turbine, leaves it as wet steam at 0.08 bar. The wet steam is condensed to saturated water state in the condenser, thus the cycle is completed.

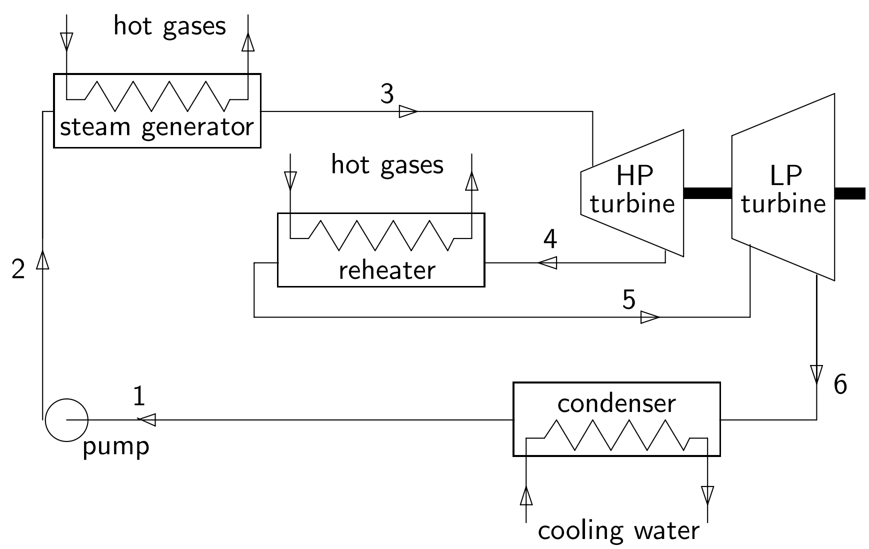


Figure 12.6 Flow diagram of a steam turbine with a reheater.

(a) Determine the enthalpies of the flow at the inlets and the outlets of the turbines and the pump.
(b) Determine the combined power production at the HP and the LP turbines, power requirement of the pump, and the back work ratio.
(c) Determine the total heat input to the steam generator and the reheater, and the thermal efficiency of the steam turbine plant.
(d) Determine the heat rejected at the condenser.

## Solution to Example 12.5

At state 1, which is the pump inlet, we have saturated water at 0.08 bar. Therefore, $h_1 = 174$ kJ/kg.

At state 2, which is the pump outlet, we have compressed water at 70 bar. Since the flow through the pump is reversible adiabatic, we get $h_2 \approx 181$ kJ/kg, as worked out in the **Solution to Example 12.4**.

At state 3, which is the HP turbine inlet, we have superheated steam at 70 bar and 450°C. Therefore, $h_3 = 3287$ kJ/kg.

At state 4, which is the HP turbine outlet, we have steam at 7 bar. Since the flow through the turbine is reversible adiabatic, $s_4 = s_3 = s$ at 70 bar and 450°C $=$ $6.632$ kJ/kg·K. For a state at 7 bar and $s_4 =$ 6.632 kJ/kg·K, the dryness fraction can be calculated using

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.632 - 1.992}{4.717} = 0.9837$$

The specific enthalpy is then

$$h_4 = h_f + x_4 \times h_{fg} = 697 + 0.9837 \times 2067 = 2730 \text{ kJ/kg}$$

At state 5, which is the LP turbine inlet, we have superheated steam at 7 bar and 425°C. Therefore, $h_5 = 3322$ kJ/kg.

At state 6, which is the LP turbine outlet, we have wet steam at 0.08 bar. Since the flow through the turbine is reversible adiabatic, $s_6 = s_5 =$ $s$ at 7 bar and 425°C $= 7.710$ kJ/kg·K. For a state at 0.08 bar and $s_6 = 7.710$ kJ/kg·K, the dryness fraction can be calculated using

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{7.710 - 0.593}{7.634} = 0.9323$$

The specific enthalpy is then

$$h_6 = h_f + x_6 \times h_{fg} = 174 + 0.9323 \times 2402 = 2413 \text{ kJ/kg}$$

Summarizing the results obtained above, we have

| State | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Condition | sat. water | comp. water | super. steam | wet steam | super. steam | wet steam |
| $P$ (in bar) | 0.08 | 70 | 70 | 7 | 7 | 0.08 |
| $T$ (in °C) | 41.5 | - | 450 | 165 | 425 | 41.5 |
| $h$ (in kJ/kg) | 174 | 181 | 3287 | 2730 | 3322 | 2413 |
| $x$ | 0 | - | - | 98.37% | - | 93.23% |

Note: 'sat.' stands for saturated, 'comp.' for compressed, and 'super.' for superheated.

(b) Using (10.9) for the flow through the adiabatic HP turbine, we have

$$\begin{aligned}[(\dot{W}_s)_{out}]_{HPturbine} &= \dot{m}(h_3 - h_4) = 50 \times (3287 - 2730) \text{ kJ/s} \\ &= 27.85 \text{ MW}\end{aligned}$$

Using (10.9) for the flow through the adiabatic LP turbine, we have

$$\begin{aligned}[(\dot{W}_s)_{out}]_{LPturbine} &= \dot{m}(h_5 - h_6) = 50 \times (3322 - 2413) \text{ kJ/s} \\ &= 45.45 \text{ MW}\end{aligned}$$

The combined power production at the HP and the LP turbines is the summation of the power productions of the HP and the LP turbines, which is 73.30 MW.

Using (10.11) for the flow through an adiabatic pump, we have

$$[(\dot{W}_s)_{in}]_{pump} = \dot{m}(h_2 - h_1) = 50 \times (181 - 174) = 0.35 \text{ MW}$$

The back work ratio is calculated as

$$\text{bwr} = \frac{\text{pump work input}}{\text{combined turbine work output}} = \frac{0.35 \text{ MW}}{73.30 \text{ MW}} = 0.5\%$$

(c) Heat input to the steam generator is calculated by applying (10.20) to the fluid flowing through the steam generator as

$$\begin{aligned}[\dot{Q}_{in}]_{steam\ generator} &= \dot{m}\,(h_3 - h_2) \\ &= 50 \times (3287 - 181) \text{ kJ/s} = 155.30 \text{ MW}\end{aligned}$$

Heat input to the reheater is calculated by applying (10.20) to the fluid flowing through the reheater as

$$\begin{aligned}[\dot{Q}_{in}]_{reheater} &= \dot{m}\,(h_5 - h_4) \\ &= 50 \times (3322 - 2730) \text{ kJ/s} = 29.60 \text{ MW}\end{aligned}$$

The total heat input to the steam generator and the reheater is the summation of the heat inputs to the steam generator and the reheater, which is 184.90 MW.

The thermal efficiency of the steam turbine plant is calculated as

$$\begin{aligned}\eta_{th} &= \frac{\text{net work out}}{\text{heat in}} \\ &= \frac{73.30 \text{ MW} - 0.35 \text{ MW}}{184.90 \text{ MW}} = \frac{72.95}{184.90} = 39.5\%\end{aligned}$$

**Comment:** Only about 39.5% of the 184.90 MW combined heat input to the steam generator and the reheater is available for electricity generation.

(d) Heat rejected at the condenser is calculated by applying (10.21) to the fluid flowing through the condenser as

$$\begin{aligned}[\dot{Q}_{out}]_{condenser} &= \dot{m}\,(h_6 - h_1) \\ &= 50 \times (2413 - 174)\ \text{kJ/s} = 111.95\ \text{MW}\end{aligned}$$

**Comment:** The percentage of the combined heat input that is rejected by the condenser is calculated using (111.95/184.90)×100, which equals 60.5%. In other words, for each MW of electric power generation there is at least about 1.5 (=111.95/72.95) MW of power is wasted. All this waste energy reaching the environment is a source of thermal pollution.

## Example 12.6

Consider the schematic of the steam turbine power plant shown in Figure 12.7. saturated water at 7 bar flowing at a rate of 50 kg/s is compressed adiabatically by a pump to 70 bar, and is fed to a steam generator where it is converted to superheated steam at 450°C at the same pressure. The superheated steam flows adiabatically through the $1^{st}$ stage turbine, and leaves it at 7 bar. A part of the steam leaving the $1^{st}$ stage turbine is fed to an open feedwater heater operated at 7 bar. (An open feedwater heater is simply a mixing chamber in which a hot fluid stream and a cold fluid stream mix with each other to form a fluid stream at intermediate temperature.) The remaining steam is fed to the $2^{nd}$ stage turbine, through which it flows adiabatically, and leaves it as steam at 0.08 bar. It is condensed to saturated water state at the same pressure in the condenser, compressed adiabatically by a second pump to 7 bar, and fed to the open feedwater heater, thus the cycle is completed.

(a) Determine the enthalpies of the flow at the inlets and the outlets of the turbines and the pumps. Assume reversible flows through the pumps and the turbines.

(b) Determine the mass flow rate of the steam diverted to the open feedwater heater from the exit of the $1^{st}$ stage turbine, assuming adiabatic conditions at the open feedwater heater.

(c) Determine the combined power production at the turbines, combined power requirement of the pumps, and the back work ratio.

(d) Determine the total heat input to the steam generator, and the thermal efficiency of the steam turbine plant.

(e) Determine the heat rejected at the condenser.

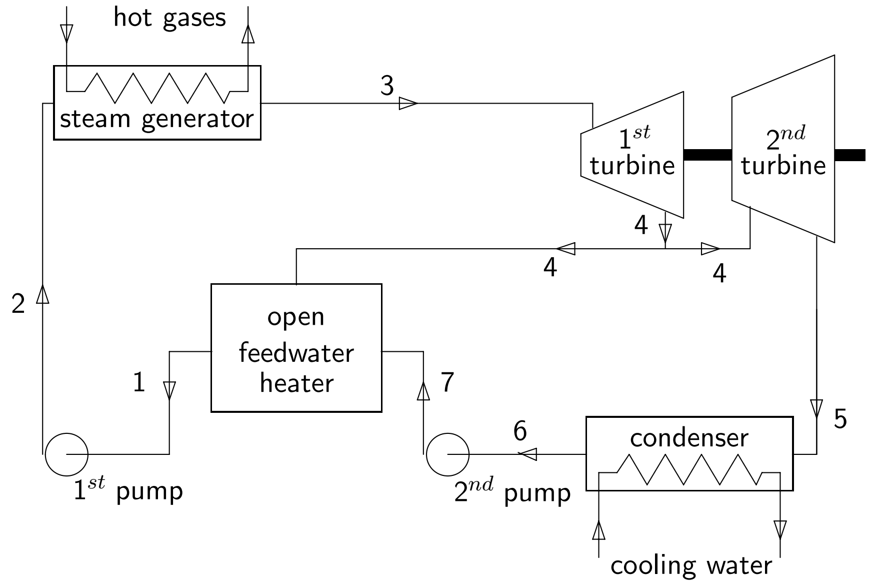


Figure 12.7 Flow diagram of a steam turbine with an open feedwater heater.

## Solution to Example 12.6

At state 1, which is the inlet of the first pump, we have saturated water at 7 bar. Therefore, $h_1 = 697$ kJ/kg.

At state 2, which is the outlet of the first pump, we have compressed water at 70 bar. Since the flow through the pump is assumed to be reversible adiabatic, $h_2$ is calculated using (12.2) as

$$h_2 \approx h_1 + v_1 (P_2 - P_1) = 697 + 0.0011082 \times (7000 - 700) = 704 \text{ kJ/kg}$$

At state 3, which is the inlet of the $1^{st}$ stage turbine, we have superheated steam at 70 bar and 450°C. At state 4, outlet of the $1^{st}$ stage turbine, we have steam at 7 bar. The flow through the turbine is assumed to be reversible adiabatic. Therefore, as found in the **Solution to Example 12.5**, $h_3 = 3287$ kJ/kg, $x_4 = 0.9837$, and $h_4 = 2730$ kJ/kg.

The wet steam leaving the 1$^{st}$ stage turbine is divided into two streams, one entering the 2$^{nd}$ stage turbine and the other entering the open feedwater heater. All these streams are assumed to be at state 4.

At state 5, which is the outlet of the 2$^{nd}$ stage turbine, we have steam at 0.08 bar. Since the flow through the 2$^{nd}$ stage turbine is reversible adiabatic, $s_5 = s_4 = s_3 = s$ at 70 bar and 450°C $= 6.632$ kJ/kg·K. For state 5 at 0.08 bar and $s_5 = 6.632$ kJ/kg·K, the dryness fraction can be calculated using

$$x_5 = \frac{s_5 - s_f}{s_g - s_f} = \frac{6.632 - 0.593}{7.634} = 0.7911$$

The specific enthalpy is then

$$h_5 = h_f + x_5 \times h_{fg} = 174 + 0.7911 \times 2402 = 2074 \text{ kJ/kg}$$

At state 6, which is the inlet of the second pump, we have saturated water at 0.08 bar. Therefore, $h_6 = 174$ kJ/kg.

At state 7, which is the outlet of the second pump, we have is compressed water at 7 bar. Since the flow through the pump is assumed to be reversible adiabatic, $h_2$ is calculated using (12.2) as

$$h_7 \approx h_6 + v_6\,(P_7 - P_6) = 174 + 0.0010084 \times (700 - 8) = 174.7 \text{ kJ/kg}$$

Summarizing the results obtained above, we have

| State | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Condition | sat. water | comp. water | super. steam | wet steam | wet steam | sat. water | comp. water |
| $P$ (in bar) | 7 | 70 | 70 | 7 | 0.08 | 0.08 | 7 |
| $T$ (in °C) | 165 | - | 450 | 165 | 41.5 | 41.5 | - |
| $h$ (in kJ/kg) | 697 | 704 | 3287 | 2730 | 2074 | 174 | 174.7 |
| $x$ | 0 | - | - | 98.37% | 79.11% | 0 | - |

Note: 'sat.' stands for saturated, 'comp.' for compressed, and 'super.' for superheated.

(b) Take the mass flow rate of the steam diverted to the open feedwater heater from the exit of the 1$^{st}$ stage turbine as $m_o$ kg/s. The steam entering the 2$^{nd}$ stage turbine is therefore (50 - $m_o$) kg/s, which is of course the same amount pumped into the open feedwater heater by the second pump.

Energy balance over the adiabatic open feedwater heater, which in effect is an adiabatic mixing chamber, is expressed by (10.16). Applying (10.16) to the adiabatic open feedwater heater of Figure 12.7, we have

$$(50 - m_o) \times (h_1 - h_7) = m_o \times (h_4 - h_1)$$

which gives

$$m_o = \frac{h_1 - h_7}{h_4 - h_7} \times 50 \text{ kg/s}$$

Substituting the known enthalpies, we get

$$m_o = \frac{697 - 174.7}{2730 - 174.7} \times 50 \text{ kg/s} = 0.2044 \times 50 \text{ kg/s} = 10.22 \text{ kg/s}$$

(c) Using (10.9) for the flow through the adiabatic 1$^{st}$ stage turbine, we have

$$\begin{aligned}[(\dot{W}_s)_{out}]_{1^{st}\,stage\,turbine} &= \dot{m}(h_3 - h_4) \\ &= 50 \times (3287 - 2730) \text{ kJ/s} \\ &= 27.85 \text{ MW}\end{aligned}$$

Using (10.9) for the flow through the adiabatic 2$^{nd}$ stage turbine, we have

$$\begin{aligned}[(\dot{W}_s)_{out}]_{2^{nd}\,stage\,turbine} &= \dot{m}(h_4 - h_5) \\ &= (50 - m_o) \times (2730 - 2074) \text{ kJ/s} \\ &= 26.10 \text{ MW}\end{aligned}$$

The combined power production of the two turbines is the summation of the power productions of the 1$^{st}$ stage and the 2$^{nd}$ stage turbine, which is = (27.85 + 26.10) MW = 53.95 MW.

Using (10.11) for the flow through an adiabatic first pump, we have

$$\begin{aligned}[(\dot{W}_s)_{in}]_{1^{st}\,pump} &= \dot{m}(h_2 - h_1) \\ &= 50 \times (704 - 697) = 0.35 \text{ MW}\end{aligned}$$

Using (10.11) for the flow through an adiabatic second pump, we have

$$\begin{aligned}[(\dot{W}_s)_{in}]_{2^{nd}\,pump} &= \dot{m}(h_7 - h_6) \\ &= (50 - m_o) \times (174.7 - 174) = 0.03 \text{ MW}\end{aligned}$$

The back work ratio is calculated as

$$\text{bwr} = \frac{\text{combined pump work input}}{\text{combined turbine work output}} = \frac{(0.35 + 0.03) \text{ MW}}{53.95 \text{ MW}} = 0.7\%$$

(d) Heat input to the steam generator is calculated by applying (10.20) to the fluid flowing through the steam generator as

$$\begin{aligned}[\dot{Q}_{in}]_{steam\ generator} &= \dot{m}\,(h_3 - h_2) \\ &= 50 \times (3287 - 704) \text{ kJ/s} = 129.15 \text{ MW}\end{aligned}$$

The thermal efficiency of the steam turbine plant is calculated as

$$\begin{aligned}
\eta_{th} &= \frac{\text{net work out}}{\text{heat in}} \\
&= \frac{\text{combined turbine work - combined pump work}}{\text{heat in}} \\
&= \frac{53.95 \text{ MW} - (0.35 + 0.03) \text{ MW}}{129.15 \text{ MW}} = \frac{53.57}{129.15} = 41.5\%
\end{aligned}$$

(e) Heat rejected at the condenser is calculated by applying (10.21) to the fluid flowing through the condenser as

$$\begin{aligned}
[\dot{Q}_{out}]_{condenser} &= \dot{m}\,(h_5 - h_6) \\
&= (50 - m_o) \times (2074 - 174) \text{ kJ/s} = 75.58 \text{ MW}
\end{aligned}$$

**Comment:** For each MW of power produced by the turbine, about 1.4 MW of power is wasted. All this waste energy reaching the environment is a source of thermal pollution.

## 12.4 Gas Turbine - Steam Turbine Combined Power Plant

One of the major drawbacks in using a gas turbine power plant for electric power generation in comparison to a steam turbine power plant is the back work ratio of the gas turbine plant (see, **Example 12.1**) being considerably higher than the back work ratio of the steam turbine power plant (see, **Example 12.4**). Nearly half of the power produced by the gas turbine is consumed by the compressor, which is used to compress the atmospheric air to a high pressure. In the steam turbine power plant, the pump, which is used to compress the water at a vacuum pressure to a high pressure, consumes less than about 1% of the power produced by the turbine.

Besides, in a gas turbine power plant, the gases produced in the burning of a fuel with air, after expanding through the turbine, are emitted into the atmosphere at a high temperature. Also, the mass flow rate of these hot exhaust gases is large owing the high air-fuel ratio used in the gas turbine. In a steam turbine power plant, on the other hand, hot gases produced by the combustion is used to superheat steam (see, Figure 12.5), which expands through the turbine causing the turbine shaft to rotate.

It is therefore advisable to combine the gas turbine and the steam turbine such that the hot exhaust gases leaving the gas turbine power plant can be used to generate the superheated steam required by the steam turbine power plant. Such is practised in power plants known as the combined gas turbine - steam turbine power plants, and the demand for which is increasing dramatically worldwide over the recent years. The basic working principle of a combined simple gas turbine - steam turbine power plant used for electric power generation is shown in the flow diagram of Figure 12.8.

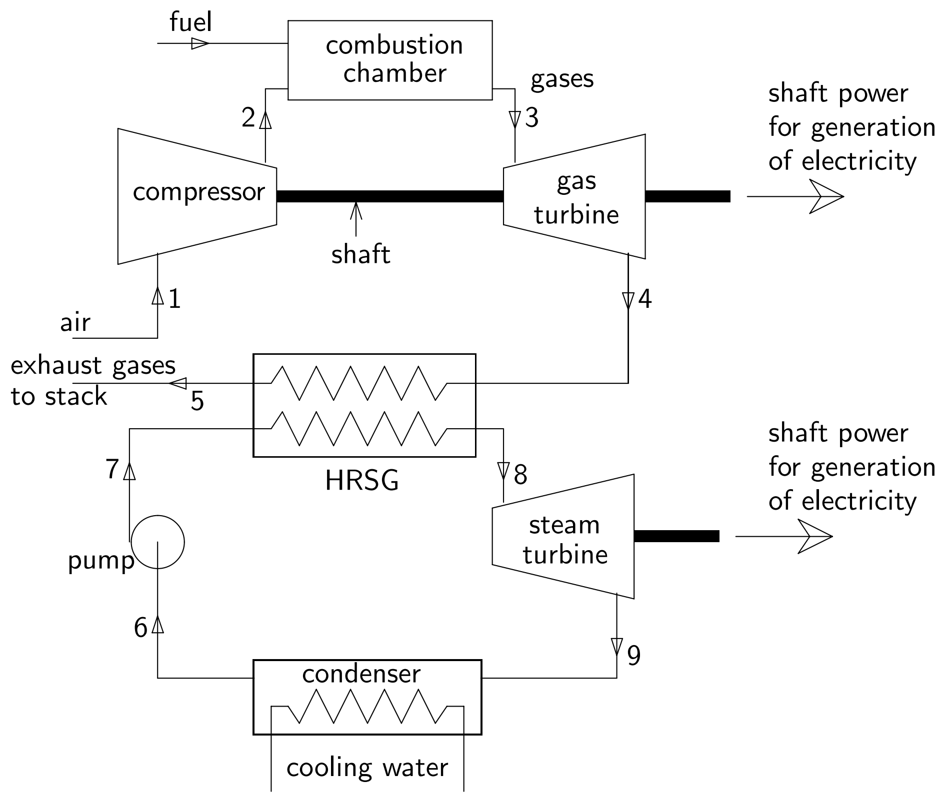


Figure 12.8 Flow diagram of a simple gas turbine - steam turbine combined power plant.

The crucial feature of a combined power plant is the heat recovery steam generator, abbreviated HRSG, shown in Figure 12.8. HRSG is basically a heat exchanger in which the hot gases leaving the gas turbine heats the compressed water supplied by the pump to superheated steam required by the steam turbine. Since the heat input to the HRSG is the heat that would have otherwise been lost to the environment, the overall thermal efficiency of a combined plant in general is high.

**Example 12.7**

Combine the gas turbine power plant of **Example 12.1** and the steam turbine power plant of **Example 12.4** using a heat recovery steam generator as shown in Figure 12.6. The mass flow rate of air through the gas turbine is 350 kg/s and the mass flow rate of water through the steam turbine is 50 kg/s. The properties at the states marked 1 to 9 in the figure are as tabulated below:

| State | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Condition | air | air | gases | gases | gases | sat. water | comp. water | super. steam | wet steam |
| $P$ (in bar) | 1 | 5 | 5 | 1 | 1 | 0.08 | 70 | 70 | 0.08 |
| $T$ (in °C) | 27 | 202.2 | 927 | 484.7 | $T_5$ | 41.5 | 41.8 | 450 | 41.5 |

Note: 'sat.' stands for saturated, 'comp.' for compressed, and 'super.' for superheated.

Assume adiabatic conditions at the HRSG and reversible adiabatic flows through the compressor, pump and the turbines. Ignore the fuel flow rate. Take $C_p$ and $\gamma$ for air and the gases as 1.005 kJ/kg · K and 1.4, respectively.

(a) Determine the combined power production at the turbines, combined power requirement of the compressor and the pump, and the back work ratio.

(b) Determine the heat input to the combined power plant, and the overall thermal efficiency of the plant.

(c) Determine the net energy rejection to the environment by the combined power plant.

(d) Determine the heat rejection by the condenser and the loss of energy to the environment with the exhaust leaving the HRSG.

(e) Determine the temperature of the exhaust gases.

## Solution to Example 12.7

(a) Data for the gas turbine power plant are the same as in **Example 12.1**, and for the steam turbine power plant are the same as in **Example 12.4**. The flows through the compressor, pump and the turbines are assumed to be reversible and adiabatic.

The compressor work input and the gas turbine power output are, therefore, the same as that are evaluated in the **Solution to Example 12.1**, which are 61.63 MW and 155.58 MW, respectively.

The pump work input and the steam turbine power output are, therefore, the same as that are evaluated in the **Solution to Example 12.4**, which are 0.35 MW and 60.65 MW, respectively.

The combined power production at the turbines is the summation of the power produced at the gas and the steam turbines, which is (155.58 + 60.65) MW = 216.23 MW

The combined power requirement of the compressor and the pump is the summation of the power inputs to the compressor and the pump, which is (61.63 + 0.35 ) MW = 61.98 MW.

The back work ratio is calculated using

$$
\begin{aligned}
\text{bwr} &= \frac{\text{pump work input} + \text{compressor work input}}{\text{combined turbine work output}} \\
&= \frac{61.98 \text{ MW}}{216.23 \text{ MW}} = 28.7\%
\end{aligned}
$$

**Comment:** About 28.7% of the combined power production of the turbines is consumed by the compressor and the pump. The remaining 71.3% is available for electricity generation. That is, the back work ratio is reduced when a gas turbine is used combined with the steam turbine.

(b) Heat is supplied to the combined power plant of Figure 12.8 only at the combustion chamber. Therefore, the heat input to the combined power plant is the same as that is evaluated in the **Solution to Example 12.1**, which is 254.95 MW.

The overall thermal efficiency of the combined power plant is calculated using

$$\begin{aligned}\eta_{th} &= \frac{\text{net work output of the combined power plant}}{\text{heat input to the combined power plant}} \\ &= \frac{216.23 \text{ MW} - 61.98 \text{ MW}}{254.95 \text{ MW}} \\ &= \frac{154.25}{254.95} = 60.5\%\end{aligned}$$

**Comment:** The thermal efficiency of the combined power plant, 60.5%, is much higher than the thermal efficiency of the gas turbine power plant, 36.9%, or the steam turbine power plant, 38.8%, when operated separately. It is the principal feature for which the combined power plant is becoming popular as a mode of electricity generation.

(c) The total heat supply to the combined power plant is 254.95 MW. The net work output of the plant is 154.25 MW. The net energy rejection to the environment by the plant is therefore calculated as (254.95 MW - 154.25 MW), which is 100.70 MW.

**Comment:** That is, 39.50% of the heat input to the combined power plant is lost to the environment. In other words, for each MW of power produced, there is about 0.65 (=100.70/154.24) MW of power wasted. This amount is far less than the amount of power wasted per MW of power produced in the cases where the gas turbine and the steam turbine are operated separately. It is yet another favourable feature that promotes the use of combined power plant for electricity generation.

(d) Energy rejection at the condenser is the same as that is evaluated in the **Solution to Example 12.4**, which is 95.00 MW. The loss of energy to the environment with the exhaust leaving the HRSG is therefore calculated as (100.70 MW - 95.00 MW), which is 5.7 MW.

(e) To determine the temperature of the exhaust gases leaving the HRSG, which is $T_5$, let us redraw the HRSG part of the combined power plant as in Figure 12.9, with the pressure and temperature data displayed on it.

Treating the HRSG as an adiabatic heat exchanger, we can write the steady flow energy equation over it to get

$$\dot{m}_{gases}\,(C_p)_{gases}\,(T_4 - T_5) = \dot{m}_{water/steam}\,(h_8 - h_7)$$

Substituting the known numerical values in the above expression, we get

$$350 \times 1.005 \times (484.7 - T_5) = 50 \times (3287 - 181)$$

where $h_8 = 3287$ kJ/kg for superheated steam at 70 bar at 450°C and $h_7 =$ 181 kJ/kg for compressed water at 70 bar at 41.8°C. Therefore, we get $T_5 =$ 43°C.

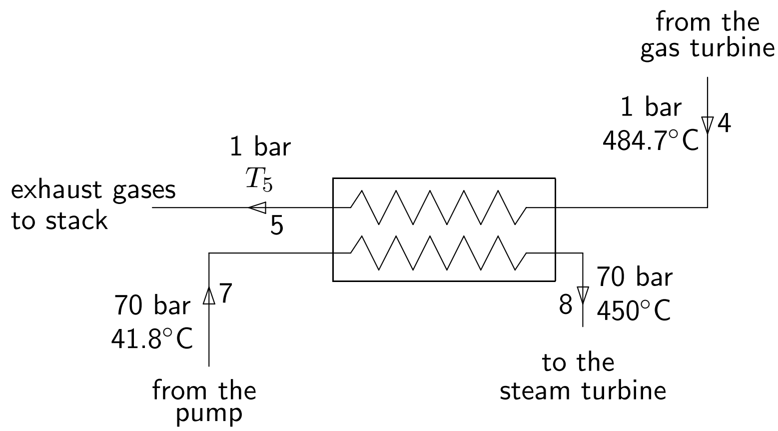


Figure 12.9 HRSG unit of the combined power plant.

**Comment:** Note that, in the HRSG, for the heat to flow from the hot gases to the water/steam mixture, $T_4$ must be greater than $T_8$ and $T_5$ must be greater than $T_7$, and these conditions are satisfied in the case considered. However, in reality, it is impractical to reduce the temperature of the exhaust gases leaving the HRSG to such low values as 43°C.

## 12.5 Minimizing the Heat Loss from Power Plants

It is of importance to note that none of the power plant studied in this chapter succeeds in converting all heat supplied to the power plant into work. That is, the thermal efficiency of no engine is 100%. Ideally, we would have liked to reach 100% thermal efficiency, since it would result in no waste of the precious heat energy, which we obtain mostly by burning the fossil fuels that generates carbon dioxide and other greenhouse gases that are responsible for global warming and the resulting climate change. Besides, the heat that is lost to the environment from the power plants causes thermal pollution.

We could have reached 100% thermal efficiency, if all heat gained by the working fluid were converted into net work. According to the first law, it is possible to convert all heat into work, since the first law states that energy cannot be created or destroyed but it can be changed from one form to another.

But, centuries of experience has shown that it is impossible to convert all heat into work in engines of the kinds studied in this chapter. This observation has been generalized into a fundamental law, which, indeed, is the second law of thermodynamics, an introduction to which is given in the next chapter.