Chapter 11

ENTROPY

> There is nothing like looking, if you want to find something. You certainly usually find something, if you look, but it is not always quite the something you were after.
>
> – J.R.R. Tolkien (*The Hobbit*)

In the preceding chapters, we have learnt many aspects of the first law applications to various thermodynamic systems. We have also learnt to use the thermodynamic properties, such as pressure, temperature, internal energy and enthalpy, when analysing thermodynamic systems. In this chapter, we will learn yet another thermodynamic property known as entropy, and its use in the thermodynamic analyses of systems.

11.1 Reversible Process

The property entropy is defined for an ideal process known as the **reversible process**. Let us therefore first see what a reversible process is all about. If we can execute a process which can be reversed without leaving any trace on the surroundings, then such a process is known as the reversible process. That is, if a reversible process is reversed then both the system and the surroundings are returned to their respective original states at the end of the reverse process. Processes that are not reversible are called irreversible processes.

`Student:` Teacher, what exactly is the difference between a reversible process and a cyclic process?

`Teacher:` In a cyclic process, the system returns to its original state. That's all. We don't bother about what happens to its surroundings when the system is returned to its original state. In a reversible process, on the other hand, the system need not return to its original state. However, the path of the reversible process must be such that if the process is reversed so as to return the system to its original state then its surroundings must also return to its original state, as though the process has never occurred in the first place.

`Student:` How to carry out a process such that, when reversed, no trace is left to indicate that the process has ever occurred?

`Teacher:` If a process is carried out in such a manner that no irreversibilities take place within or outside the boundaries of the system then it is possible to reverse the system and the surroundings to their respective original states as though the process has never occurred.

`Student:` Is it possible to realize such a process?

`Teacher:` Of course, it is not possible to realize a reversible process in real life situations. But, if we can minimize the irreversibilities occurring within and outside the boundary of a system during a process then such a process can be said to approach a reversible process.

`Student:` What do you mean by irreversibilities, Teacher?

`Teacher:` Friction is an irreversibility. During a process, some amount of work may be done to overcome friction force. If such a process is reversed, we will not be able to take back the energy used as work to overcome friction force. As a matter of fact, some more work has to be done to overcome friction during the reversal of the process as well. Such a process is far from a reversible process. If we are to approach a reversible process then the process taking place must experience very little or no friction forces.

`Student:` Okay, Teacher. I see that friction is an irreversibility. That, I must say, is very easy to understand and accept. Are there any other forms of irreversibilities, Teacher?

`Teacher:` Yes, there are. Unrestrained expansion or compression is another form of irreversibility. A reversible expansion or compression process is imagined to take place under infinitely small pressure difference between the system and its surroundings, without destroying the uniform distribution of the properties within the system. That means, during a reversible expansion or compression process, the system passes through a series of equilibrium states. Therefore, it is possible to reverse it at any time during the process without leaving a trace on the surroundings.

`Student:` Oh.. you are talking about the quasistatic process that we have learnt in Chapter 7.

`Teacher:` Yes, a quasistatic process carried out under certain conditions is an example of a reversible process.

`Student:` What are those conditions, Teacher?

`Teacher:` A quasistatic process carried out under adiabatic conditions is indeed a reversible process.

`Student:` Why do we require the adiabatic condition, Teacher?

`Teacher:` Because heat transfer between a system and its surroundings is always an irreversible process. You know that heat is transferred from a system to its surroundings only if the temperature of the system is higher than the temperature of the surroundings. If you are to reverse this process, then heat must be transferred from the low temperature surroundings to the high temperature system, which is impossible.

`Student:` Okay, Teacher. Are you saying that a reversible process must always be an adiabatic process?

`Teacher:` Well, we can always imagine a non-adiabatic process where the heat transfer between the system and its surroundings takes place under the condition of extremely small temperature difference between the system and its surroundings. Such a process may be said to approach a reversible process even though it is not an adiabatic process.

`Student:` Okay, Teacher. Let me see. If there is a non-adiabatic quasistatic process, then heat is transferred between the system and the surroundings under conditions of extremely small temperature differences between the system and the surroundings. Is that correct?

`Teacher:` Yes, that is correct.

`Student:` Teacher, don't you think it would be very hard to transfer heat across extremely small temperature differences?

`Teacher:` Yes, it would be impractical since it would take a very long time to transfer heat across very small temperature differences between the system and its surroundings.

`Student:` Teacher, I understand from what you are saying that a reversible process is a highly idealized process, and that it is next to impossible to execute a reversible process in reality. Why do we study it then?

`Teacher:` We study about reversible processes for two reasons. First, they are easy to analyze since a system passes through a series of equilibrium states during a reversible process. Second, they give the Nature's limit imaginable for the corresponding real processes. Reversible processes can be viewed as theoretical limits for the corresponding irreversible ones.

11.2 Definition of Entropy

The property entropy is defined in terms of change in entropy as

$$dS \equiv \left(\frac{dQ_{in}}{T}\right)_{rev} \tag{11.1}$$

where the label "rev" stands for reversible process.

Integrating the above, we get

$$\Delta S = \int_{S_o}^{S_f} dS = \int_{o}^{f} \frac{(dQ_{in})_{rev}}{T} \tag{11.2}$$

which could be used for the evaluation of entropy change for a reversible process between state o and state f.

In this textbook, we use S to denote entropy, and s to denote both the specific and molar entropies. The unit kJ/K is used for entropy, kJ/kg · K for specific entropy, and kJ/kmol · K for molar entropy.

11.3 Evaluation of Entropy Change for Any Process

Entropy change could be evaluated using (11.2) provided the process concerned is a reversible process. In this section, we will obtain expressions that could be used to evaluate the entropy changes of both reversible and irreversible processes.

Let us start from (11.1), which could be rearranged to give

$$T dS = (dQ_{in})_{rev} \tag{11.3}$$

Using the first law, the dQ_{in} term in (11.3) can be replaced to give

$$T dS = (dU - dW_{in})_{rev} \tag{11.4}$$

Since the internal energy change dU is independent of the path of the process, but dW_{in} depends on the path of the process, (11.4) can be written as

$$T dS = dU - (dW_{in})_{rev} \tag{11.5}$$

The work term in a reversible process of a simple compressible system is the boundary work only. Since the path of a reversible process consists of a series of equilibrium states, the boundary work involved in a reversible process shall be given by (7.5). Using (7.5) in (11.5), we get

$$T dS = dU + P\, dV \tag{11.6}$$

which can be written in terms of specific (or molar) properties as

$$T ds = du + P\,dv \tag{11.7}$$

Using $h = u + Pv$ in (11.7) to eliminate du, we get

$$T ds = dh - v\,dP \tag{11.8}$$

Equations (11.7) and (11.8) are the most general equations applicable to evaluate the entropy change of any system undergoing any process, reversible or irreversible.

Upon rearranging and integrating (11.7), we get

$$\Delta s = \int_{u_o}^{u_f} \frac{du}{T} + \int_{v_o}^{v_f} \frac{P\,dv}{T} \tag{11.9}$$

Upon rearranging and integrating (11.8), we get

$$\Delta s = \int_{h_o}^{h_f} \frac{dh}{T} - \int_{P_o}^{P_f} \frac{v\,dP}{T} \tag{11.10}$$

Equations (11.9) and (11.10) could be used to determine the entropy change of any process between state o and state f for any substance.

`Student:` Teacher, could you tell me why you dropped the subscript $_{rev}$ from (11.6) which you derived from (11.5), which is valid only for a reversible process. Is that a mistake?

`Teacher:` No, it is not a mistake. Observe that each term in (11.6) is a property, and the differential of a property depends only on the values of the properties at the initial and the final states, not on the path of the process. Therefore, (11.6), (11.7) and (11.8) are applicable for any process, reversible or not. But, dW_{in} in (11.5) is a path dependent function, and therefore it is necessary to specify the nature of the process for which (11.5) can be applied. The same can be said about (11.3) where the path dependent function dQ_{in} is equated to a combination of properties, $T\,dS$.

`Student:` Teacher, I can't agree with what you say.

`Teacher:` Why not?

`Student:` Take the first law of thermodynamics given by $dQ_{in}+dW_{in} = dU$, in which the summation of two path dependent functions dQ_{in} and dW_{in} are equated to the differential in the property U. But, we know that we can apply $dQ_{in} + dW_{in} = dU$ to any process as far as the system concerned is a closed system.

`Teacher:` When adding two path dependent functions, it is possible that the summation becomes independent of the path as in the case of the first law. So that the first law is applicable for any process. But, in (11.3) or in (11.5), there is only one path dependent function against a combination of properties, which can be equated to each other only under special circumstances, such as a reversible process.

`Student:` Oh... I see.

`Teacher:` It is also of interest to note that even though (11.7), and (11.8) are obtained using the first law applied to closed system, these equations are applied to fluid flowing through open systems as well since these equations contain only properties.

11.4 Isentropic Process

For an adiabatic process, $dQ_{in} = 0$. If the adiabatic process considered is a reversible process then (11.1) will give $dS = 0$. That is to say entropy remains constant for a **reversible, adiabatic process**. Such a process is therefore known as an **isentropic process**.

11.4.1 Isentropic Process of an ideal gas

For an isentropic process, $dS = 0$. Therefore, (11.6) reduces to

$$dU + P\,dV = 0 \tag{11.11}$$

Since $dU = m\,C_v\,dT$ for an ideal gas, (11.11) can be rewritten as

$$m\,C_v\,dT + P\,dV = 0 \tag{11.12}$$

which gives

$$-P\,dV = m\,C_v\,dT \tag{11.13}$$

which is the same as (7.27), the equation from which (7.29), (7.30) and (7.31) describing the quasistatic adiabatic process, which is indeed an isentropic process, of an ideal gas are derived. So, whenever we use (7.29), (7.30) or (7.31) to describe a process of an ideal gas, we in fact are using the information that entropy remains constant for that process.

We have already worked out problems involving the quasistatic adiabatic processes of ideal gas (that is, isentropic processes of ideal gas) in the preceding chapters. We made use of (7.29), (7.30) or (7.31) to work out such problems.

11.4.2 Isentropic Process of Steam/Water Systems

Entropy of a given mass of steam/water remains constant throughout an isentropic process. We will therefore use the information that the specific entropy of a system remains constant when it undergoes an isentropic process to refer to the steam table to determine the required equilibrium state of the system along the isentropic process considered.

In this chapter, we shall work out problems involving the isentropic processes of steam/water systems which we have not dealt with so far in this book.

11.5 Worked Examples

Example 11.1

Based on equation (11.1), is it reasonable to conclude that the entropy of a closed system remains constant during an adiabatic process?

Solution to Example 11.1

Equation (11.1) is applicable for a reversible process only. If the given adiabatic process is reversible then $dS = 0$, which means that entropy remains constant for a reversible adiabatic process.

But, if the given adiabatic process is not a reversible process, then (11.1) is not applicable for that process. Therefore, dS may not be equal to zero. And, thus, we cannot say that entropy remains constant for such a process.

Example 11.2

Is the numerical value of the integral $\int_1^2 dQ/T$ the same for all processes between states 1 and 2?

Solution to Example 11.2

For a reversible process,

$$\int_1^2 dQ/T = \Delta S = S_2 - S_1$$

Since the difference (S_2 - S_1) depends only on the states 1 and 2, $\int_1^2 dQ/T$ remains the same for all reversible processes between states 1 and 2.

If the process considered is not a reversible one, then $\int_1^2 dQ/T$ cannot be equated to (S_2 - S_1), and therefore we cannot expect the numerical value of the integral $\int_1^2 dQ/T$ to be the same for all processes between states 1 and 2.

Example 11.3

Show that, for a reversible isothermal process, the net heat supply to the system from its surroundings can be evaluated using $Q_{in} = T\,\Delta S$.

Solution to Example 11.3

For a reversible process, (11.2) can be applied. If the given process is a reversible isothermal process, then T is a constant in (11.2) and therefore T can be taken out of the integral sign in (11.2). Thus, we get

$$\Delta S = \frac{1}{T}\int_o^f (dQ_{in})_{rev} = \frac{Q_{in}}{T} \tag{11.14}$$

where Q_{in} is the net heat supplied to the system undergoing a reversible isothermal process between states o and f. Equation (11.14) can be rearranged to give

$$Q_{in} = T\,\Delta S \quad \text{for a reversible isothermal process.} \tag{11.15}$$

Example 11.4

Saturated steam at 40 bar expands reversibly and isothermally to 5 bar in a piston-cylinder assembly. The mass of steam is 5 kg. Determine the heat and work exchanges of the steam with its surroundings.

Solution to Example 11.4

Temperature of saturated steam at 40 bar can be found from the Saturated Steam Table as 250.3°C. Since the expansion is reversible and isothermal, (11.15) could be used to determine the heat exchange between steam and its surroundings.

Initially, steam is at saturated vapour state at 40 bar. Specific entropy at the initial state can therefore be found from the Steam Table as 6.070 kJ/kg · K.

Since the expansion process takes place isothermally, temperature at the final state remains at 250.3°C. Pressure at the final state is 5 bar. Specific entropy at the final state can therefore be found from the Steam Table as ≈7.271 kJ/kg · K.

The total entropy difference of the given isothermal process is then given by

$$\Delta S = (5\ \text{kg}) \times (7.271 - 6.070)\ \text{kJ/kg}\cdot\text{K} = 6.005\ \text{kJ/K}$$

Using the above in (11.15), we get

$$Q_{in} = (273 + 250.3)\ \text{K} \times 6.005\ \text{kJ/K} = 3142.4\ \text{kJ}$$

The work output could be calculated using the first law as

$$W_{out} = Q_{in} - \Delta U = 3142.4 \text{ kJ} - (5 \text{ kg}) \times (u_f - u_o)$$

where $u_f = u$ for steam at 5 bar at 250.3°C $\approx$ 2725 kJ/kg and $u_o = u$ for saturated steam at 40 bar = 2602 kJ/kg.

Therefore, we get

$$W_{out} = 3142.4 \text{ kJ} - (5 \text{ kg}) \times (2725 - 2602) \text{ kJ/kg} = 2527.4 \text{ kJ}$$

Example 11.5

If the working fluid of **Example 11.4** were an ideal gas instead of steam, then the heat supplied to the working fluid would be the same as the work done by the working fluid during the expansion. Why is this not so for steam as seen in **Solution to Example 11.4**?

Solution to Example 11.5

For an ideal gas, internal energy is a function of temperature alone. Therefore, $\Delta U = 0$ for the isothermal process of an ideal gas. Thus, the first law would give that the heat supplied to the ideal gas is the same as the work done by the ideal gas during the expansion.

For steam, internal energy is not a function of temperature alone. Therefore, ΔU could take a finite value for an isothermal process of steam, as seen in the **Solution to Example 11.4**. Owing to the finite value that ΔU takes, heat supplied to steam cannot be the same as the work done by steam during its expansion.

Example 11.6

Starting from $T\, ds = dh - v\, dP$, show that for a constant-pressure process, $\Delta s = C_p \ln(T_f/T_o)$ where C_p is the specific heat at constant-pressure, taken as a constant.

Solution to Example 11.6

Since $dP = 0$ for a constant-pressure process, $T\,ds = dh - v\,dP$ reduces to

$$T\,ds = dh \quad \text{for a constant-pressure process.} \tag{11.16}$$

The enthalpy change for any substance can be written as $dh = C_p\,dT$ for a constant-pressure process of any substance (see Section 5.5). Equation (11.16) therefore becomes

$$T\,ds = C_p\,dT \quad \text{for a constant-pressure process,}$$

which upon integration gives

$$\Delta s = \int_{T_o}^{T_f} \frac{C_p}{T}\,dT \quad \text{for a constant-pressure process.} \tag{11.17}$$

When C_p is taken as a constant, (11.17) reduces to

$$\Delta s = C_p \ln\left(\frac{T_f}{T_o}\right) \quad \text{for a constant-pressure process.} \tag{11.18}$$

Equations (11.17) and (11.18) are applicable for both the reversible and irreversible constant-pressure processes of any substance.

Example 11.7

Starting from $T\,ds = du + P\,dv$, show that for a constant-volume process, $\Delta s = C_v \ln(T_f/T_o)$ where C_v is the specific heat at constant-volume, taken as a constant.

Solution to Example 11.7

Since $dv = 0$ for a constant-volume process, $T\,ds = du + P\,dv$ reduces to

$$T\,ds = du \quad \text{for a constant-volume process.} \tag{11.19}$$

The internal energy change for any substance can be written as $du = C_v\,dT$ for a constant-volume process of any substance (see Section 5.4). Equation (11.19) therefore becomes

$$T\,ds = C_v\,dT \quad \text{for a constant-volume process,}$$

which upon integration gives

$$\Delta s = \int_{T_o}^{T_f} \frac{C_v}{T}\, dT \quad \text{for a constant-volume process.} \tag{11.20}$$

When C_v is taken as a constant, (11.20) reduces to

$$\Delta s = C_v \ln\left(\frac{T_f}{T_o}\right) \quad \text{for a constant-volume process.} \tag{11.21}$$

Equations (11.20) and (11.21) are applicable for both the reversible and irreversible constant-volume processes of any substance.

Example 11.8

Show that $\Delta S = m\,C \ln(T_f/T_o)$ for an incompressible substance undergoing any process, where m is the mass of substance and C is the specific heat, taken as constant.

Solution to Example 11.8

An incompressible substance does not experience any volume change, and therefore $dv = 0$. The internal energy change for any substance undergoing no volume change can be written as $du = C_v\, dT$, where C_v is the specific heat at constant-volume (see Section 5.4). These facts about incompressible substance are the same as those for the constant-volume process considered in the **Solution to Example 11.7**. Therefore, (11.20) is applicable also for an incompressible substance undergoing any process.

For an incompressible substance, C_p and C_v take nearly the same value and we denote that value as C. Taking C as a constant, we could obtain an expression for the total entropy change, staring from (11.20), as

$$\Delta S = m\,C \ln\left(\frac{T_f}{T_o}\right) \tag{11.22}$$

for an incompressible substance undergoing any process.

Example 11.9

The temperature of a metal block of 500 kg and 0.42 kJ/kg · K specific heat is reduced from 800 K to 450 K. Determine the total entropy change of the metal block.

Solution to Example 11.9

Assuming the metal block to be an incompressible substance, we could use (11.22) to evaluate the entropy change of the metal block as follows:

$$\Delta S = (500 \text{ kg}) \times (0.42 \text{ kJ/kg} \cdot \text{K}) \times \ln\left(\frac{450}{800}\right) = -120.8 \text{ kJ/K}$$

Example 11.10

Starting from $T ds = du + P\,dv$, show that an ideal gas (with constant C_v and C_p) undergoing any process satisfies the following equations:

$$\begin{aligned} \Delta s &= C_v \ln(T_f/T_o) + R \ln(v_f/v_o) \\ &= C_p \ln(T_f/T_o) - R \ln(P_f/P_o) \\ &= C_p \ln(v_f/v_o) + C_v \ln(P_f/P_o) \end{aligned}$$

Solution to Example 11.10

For an ideal gas, $du = C_v\,dT$ and $P = RT/v$. Substituting these in $T ds = du + P\,dv$ and integrating, we get

$$\begin{aligned} \Delta s &= \int_{T_o}^{T_f} \frac{C_v\,dT}{T} + \int_{v_o}^{v_f} \frac{R\,T\,dv}{v\,T} \\ &= C_v \ln\left(\frac{T_f}{T_o}\right) + R \ln\left(\frac{v_f}{v_o}\right) \qquad (11.23) \end{aligned}$$

Using $h = u + P\,v$ in $T ds = du + P\,dv$, we can get $T ds = dh - v\,dP$ as in Section 11.3. For an ideal gas, $dh = C_p\,dT$ and $v = RT/P$. Substituting these in $Tds = dh - v\,dP$ and integrating, we get

$$\begin{aligned}\Delta s &= \int_{T_o}^{T_f} \frac{C_p\,dT}{T} - \int_{P_o}^{P_f} \frac{R\,T\,dP}{P\,T} \\ &= C_p \ln\left(\frac{T_f}{T_o}\right) - R \ln\left(\frac{P_f}{P_o}\right) \qquad (11.24)\end{aligned}$$

Multiplying (11.23) by C_p and (11.24) by C_v and subtracting one from the other, we get

$$(C_p - C_v)\,\Delta s = C_p\,R \ln\left(\frac{v_f}{v_o}\right) + C_v\,R \ln\left(\frac{P_f}{P_o}\right)$$

Since $C_p - C_v = R$, the above reduces to

$$\Delta s = C_p \ln\left(\frac{v_f}{v_o}\right) + C_v \ln\left(\frac{P_f}{P_o}\right) \qquad (11.25)$$

Example 11.11

Air is compressed from 1 bar and 300 K to 7 bar at a steady flow rate of 9 kg/s. Determine the power required if the flow through the compressor is reversible and adiabatic.

Solution to Example 11.11

Work input to the adiabatic compressor can be calculated using (10.11) provided the potential energy and kinetic energy changes across the compressor are neglected. Assuming air behaves as an ideal gas with constant C_p, (10.11) can be written as

$$\begin{aligned}(\dot{W}_s)_{in} &= \dot{m}\,C_p\,(T_e - T_i) \\ &= (9\text{ kg/s}) \times (1.005\text{ kJ/kg}\cdot\text{K}) \times (T_e - 300\text{ K}) \qquad (11.26)\end{aligned}$$

where T_e is not known.

Since the flow is reversible and adiabatic (that is, isentropic), (7.31) could be used to determine T_e as follows:

$$T_e = T_i\,\left(\frac{P_e}{P_i}\right)^{(\gamma-1)/\gamma} = 300\text{ K} \times \left(\frac{7}{1}\right)^{(1.4-1)/1.4} = 523\text{ K}$$

Therefore, (11.26) gives $(\dot{W}_s)_{in} = 2.0$ MW.

Comment: Note that we could have also calculated T_e using (11.24) in which $\Delta s = 0$ since the process is reversible and adiabatic, that is, isentropic.

Example 11.12

Air is compressed from 1 bar and 300 K to 7 bar at a steady flow rate of 9 kg/s. Determine the power required if the flow through the compressor is reversible and isothermal.

Solution to Example 11.12

Work input to a non-adiabatic compressor can be calculated using (10.10) provided the potential energy change across the compressor is negligible. When also the kinetic energy change across the compressor is neglected, (10.10) reduces to

$$\dot{Q}_{in} + (\dot{W}_s)_{in} = \dot{m}\,(h_e - h_i) \tag{11.27}$$

Since the flow through the compressor is assumed to be reversible and isothermal, we could use (11.15), which for a fluid flowing steadily at a mass flow rate of $\dot{m}$ becomes

$$\dot{Q}_{in} = \dot{m}\,T\,\Delta s = \dot{m}\,T\;(s_e - s_i) \tag{11.28}$$

Combining (11.27) and (11.28), we get

$$(\dot{W}_s)_{in} = \dot{m}\,[(h_e - h_i) - T\,(s_e - s_i)] \tag{11.29}$$

for a reversible isothermal flow through a compressor.

Integrating (11.8) for an isothermal process, we get

$$T\,(s_e - s_i) = (h_e - h_i) - \int_{P_i}^{P_e} v\,dP \tag{11.30}$$

Combining (11.29) and (11.30), we get

$$(\dot{W}_s)_{in} = \dot{m}\int_{P_i}^{P_e} v\,dP \tag{11.31}$$

for a reversible isothermal flow through the compressor.

If we assume that air behaves as an ideal gas then we get

$$(\dot{W}_s)_{in} = \dot{m}\,R\,T\,\ln\left(\frac{P_e}{P_i}\right) \tag{11.32}$$

for reversible isothermal flow of an ideal gas through a compressor.

Substituting the known values in (11.32), we get

$$(\dot{W}_s)_{in} = (9\ \mathsf{kg/s}) \times \frac{8.314}{29}\ \mathsf{kJ/kg \cdot K} \times 300\ \mathsf{K} \times \ln\left(\frac{7}{1}\right) = 1.5\ \mathsf{MW}$$

Example 11.13

Show that if the flow through a pump is assumed to be reversible then the power input to the pump can be determined, neglecting potential and kinetic energy changes across the pump, using

$$(\dot{W}_s)_{in} \approx \dot{m}\,v_i\,(P_e - P_i)$$

where i and e denote inlet and exit, respectively.

Solution to Example 11.13

Work input to a pump can be calculated using (10.10) provided the potential energy change across the pump is negligible. When also the kinetic energy change across the pump is neglected, (10.10) reduces to

$$\dot{Q}_{in} + (\dot{W}_s)_{in} = \dot{m}\,(h_e - h_i) \tag{11.33}$$

Since the flow through the pump is assumed to be reversible, we could use (11.3), which for a fluid flowing steadily at a mass flow rate of $\dot{m}$ becomes

$$d(\dot{Q}_{in})_{rev} = \dot{m}\,T\,ds$$

Upon integration, it gives

$$(\dot{Q}_{in})_{rev} = \dot{m}\int_{s_i}^{s_e} T ds \tag{11.34}$$

For a reversible flow through the pump, the heat input term in (11.33) can be substituted by the heat input term given by (11.34). We then get

$$(\dot{W}_s)_{in} = \dot{m}\left(h_e - h_i - \int_{s_i}^{s_e} Tds\right) \tag{11.35}$$

Upon integration, (11.8) gives

$$\int_{s_i}^{s_e} Tds = h_e - h_i - \int_{P_i}^{P_e} vdP \tag{11.36}$$

Using (11.36), we can reduce (11.35) to

$$(\dot{W}_s)_{in} = \dot{m}\int_{P_i}^{P_e} vdP \tag{11.37}$$

which is applicable for a reversible flow through a pump.

Since only liquids are compressed using pumps and since the specific volume of a liquid varies only slightly from the inlet to the exit of the pump, (11.37) can be approximated to

$$(\dot{W}_s)_{in} \approx \dot{m}\, v_i\, (P_e - P_i) \tag{11.38}$$

Example 11.14

A pump is used to increase the pressure of 12 kg/s of saturated water at 0.07 bar entering the pump to 15 bar with only a slight temperature increase. Determine the power input to the pump assuming reversible flow through the pump.

Solution to Example 11.14

Since the flow through the pump is assumed to be reversible, (11.38) may be used to evaluate the power input to the pump. The specific volume of saturated water at 0.07 bar entering the pump v_i is 0.0010074 m^3/kg. The mass flow rate is 12 kg/s, $P_i = 0.07$ bar and P_e is 15 bar. Substituting the known numerical values in (11.38), we get

$$(\dot{W}_{in})_{pump} \approx 12 \text{ kg/s} \times 0.0010074 \text{ m}^3\text{/kg} \times (1500 - 7) \text{ kPa} = 18.05 \text{ kW}$$

Example 11.15

Steam enters a turbine at 70 bar and 450°C and leaves it as wet steam at 0.08 bar. Steam flow rate through the turbine is 50 kg/s. Assuming the flow through turbine as reversible adiabatic, determine the work output of the turbine.

Solution to Example 11.15

At the turbine inlet, we have superheated steam 'at 70 bar and 450°C. The specific enthalpy found from a Superheated Steam Table at the turbine inlet condition is $h_i = 3287$ kJ/kg.

At the turbine outlet, we have wet steam at 0.08 bar. Since the flow through the turbine is assumed to be reversible adiabatic, the entropy of the flow remains constant. That is,

$$s_e = s_i = s \text{ at 70 bar and 450°C} = 6.632 \text{ kJ/kg} \cdot \text{K}$$

The dryness fraction at the turbine outlet can be calculated using

$$x_e = \frac{s_e - s_f}{s_{fg}} = \frac{6.632 - 0.593}{7.634} = 0.7911$$

The specific enthalpy at the turbine outlet is then

$$h_e = h_f + x_e \times h_{fg} = 174 + 0.7911 \times 2402 = 2074 \text{ kJ/kg}$$

The work output of the turbine is calculated, using (10.9) applicable for flow through an adiabatic turbine, as follows:

$$(\dot{W}_s)_{out} = \dot{m}(h_i - h_e) = 50 \times (3287 - 2074) \text{ kJ/s} = 60.65 \text{ MW}$$

Example 11.16

A rigid cylinder contains a 'floating' piston. Initially, it divides the cylinder in half, and on each side of the piston the cylinder holds 0.01 kmol of steam at 200°C and 100 kPa. Using an electrical heater installed on side A of the cylinder, heat is added slowly until the pressure in side A reaches 150 kPa. The piston and the cylinder are perfect thermal insulators with negligible heat capacity. Determine the following:

(a) Final volume of steam on side B.
(b) Work done by the piston on steam on side B.
(c) Amount of heat supplied by the electrical heater.

Solution to Example 11.16

Heat added slowly to side A causes the steam on side A to expand pushing the 'floating' piston outwards to compress the steam on side B. Both the expansion of steam on side A and the compression of steam on side B may be assumed to be quasistatic processes. Since the piston and the cylinder are perfect heat insulators, the steam on side B is assumed to undergo an adiabatic process.

Taking the molar mass of the steam as 18 kg/kmol, we find that the mass of steam in each side is m = 0.18 kg. Figure 11.1 shows all the data for the problem.

Note that this example is very similar to **Example 7.24**, except for the difference the working fluid is ideal gas in **Example 7.24** and it is steam in this example. Since the behaviour of steam in general may not be approximated to ideal gas behaviour, we cannot use (7.29), or its equivalent (7.30) or (7.31), to describe the quasistatic adiabatic process executed by steam on side B, as we did in the **Solution to Example 7.24**. Instead we shall use the fact the entropy remains constant for a quasistatic adiabatic (that is, an isentropic) process.

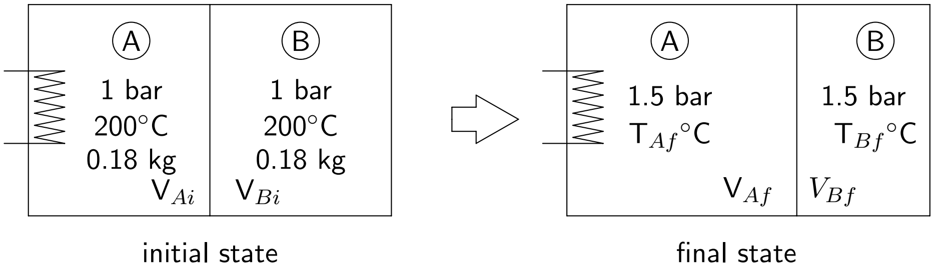

Figure 11.1 Initial and final states of Example 11.16.

(a) Determine the final volume of steam on side B.

Initially, steam on side B is at 1 bar and 200°C. From a Steam Table, we can find that the steam is at superheated vapour state with the following properties:

v_{Bi} = 2.173 m^3/kg, u_{Bi} = 2659 kJ/kg and s_{Bi} = 7.834 kJ/kg·K.

At the final state, the steam on side B is at 1.5 bar. Since the entropy and the mass of steam on side B remain constant, the specific entropy of steam on side B at the final state is the same as s_{Bi} given above.

From a Steam Table, we can find that the final state of steam on side B at 1.5 bar 7.834 kJ/kg·K specific entropy is also a superheated vapour state with the following properties:

v_{Bf} = 1.594 m^3/kg, u_{Bf} = 2729 kJ/kg and T_{Bf} = 248°C.

Since the mass of steam on side B is 0.18 kg, we get the final volume of the steam on side B as V_{Bf} = 0.287 m^3.

(b) Determine the work done by the piston on steam on side B.

Since the steam on side B undergoes an adiabatic process, the first law of thermodynamics applied to it gives

$$\begin{aligned}(W_{in})_B &= (\Delta U)_B = m\,(u_{Bf} - u_{Bi}) \\ &= 0.18 \times (2729 - 2659)\ \text{kJ} = 12.6\ \text{kJ} \qquad (11.39)\end{aligned}$$

(c) Determine the amount of heat supplied by the electrical heater.

Following the discussion on part (c) of the **Solution to Example 7.24**, we can write the heat supplied by the heater as

$$(Q_{in})_A = (\Delta U)_A + (W_{in})_B \qquad (11.40)$$

in which $(\Delta U)_A$ must be determined using

$$(\Delta U)_A = m(u_{Af} - u_{Ai}) = 0.18 \times (u_{Af} - 2659)\ \text{kJ}$$

since $u_{Ai} = u_{Bi}$.

In order to find u_{Af}, we know one property of the steam on side A at the final state, which is the pressure 1.5 bar, and we require another property which could only be the specific volume expressed as

$$v_{Af} = \frac{V_{Af}}{0.18\ \text{kg}} = \frac{V_{Ai} + V_{Bi} - V_{Bf}}{0.18\ \text{kg}}$$

in which we know $V_{Bf} = 0.287$ m^3. Since $V_{Ai} = V_{Bi}$ and since $v_{Bi} = 2.173$ m^3/kg, the above equation gives

$$v_{Af} = \frac{2 \times 0.18 \times 2.173 - 0.287}{0.18 \text{ kg}} = 2.752 \text{ m}^3/\text{kg}$$

It can be found from a Steam Table that steam at 1.5 bar and 2.752 m^3/kg specific volume is at a superheated vapour state, and the corresponding properties are

$u_{Af} = 3332$ kJ/kg and $T_{Af} = 618$°C.

Using the above value of u_{Af} and the value of $(W_{in})_B$ from (11.39) in (11.40), we get

$$(Q_{in})_A = [0.18 \times (3332 - 2659) + 12.6] \text{ kJ} = 133.7 \text{ kJ}$$

which is the heat supplied by the electrical heater.

In summary, we conclude that the 133.7 kJ of heat supplied by the electrical heater to the steam on side A, while maintaining the quasistatic nature of the process on side A, is used to raise its temperature to 618°C and for this steam to push the piston outwards by doing 12.6 kJ of work on the piston. The piston moves without friction to use the 12.6 kJ of work to quasistatically and adiabatically compress the steam on side B. The work done on this steam raises its temperature to 248°C.

11.6 Summary

- If we can execute a process which can be reversed without leaving any trace on the surroundings, then such a process is known as the reversible process. That is, if a reversible process is reversed then both the system and the surroundings are returned to their respective original states at the end of the reverse process. Processes that are not reversible are called irreversible processes.

- Entropy is defined as
$$dS \equiv \left(\frac{dQ_{in}}{T}\right)_{rev} \tag{11.1}$$

- For any system undergoing any process,
$$T\,ds = du + P\,dv \tag{11.7}$$
$$T\,ds = dh - v\,dP \tag{11.8}$$

- Entropy remains a constant for a reversible adiabatic process, which is also known as an **isentropic process**.

- The path of an isentropic process of an ideal gas can be described by (7.29), (7.30) and (7.31).

- The path of an isentropic process of a water/steam system may be traced by using the fact that entropy remains constant along such a path.

- For a reversible isothermal process of any substance,
$$Q_{in} = T\,\Delta S \tag{11.15}$$

- For both the reversible and irreversible constant-pressure processes of any substance,
$$\Delta s = \int_{T_o}^{T_f} \frac{C_p}{T}\,dT \tag{11.17}$$
which, when C_p is taken as a constant, reduces to
$$\Delta s = C_p \ln\left(\frac{T_f}{T_o}\right) \tag{11.18}$$

- For both the reversible and irreversible constant-volume processes of any substance,
$$\Delta s = \int_{T_o}^{T_f} \frac{C_v}{T}\,dT \tag{11.20}$$
which, when C_v is taken as a constant, reduces to
$$\Delta s = C_v \ln\left(\frac{T_f}{T_o}\right) \tag{11.21}$$

- For an incompressible substance, with constant specific heat C, undergoing any process,
$$\Delta s = C \ln\left(\frac{T_f}{T_o}\right) \tag{11.22}$$
where $C = C_v \approx C_p$ for an incompressible substance.

- For an ideal gas, with constant C_p and C_v, undergoing any process,

$$\begin{aligned}\Delta s &= C_v \ln\left(\frac{T_f}{T_o}\right) + R \ln\left(\frac{v_f}{v_o}\right) && (11.23)\\ &= C_p \ln\left(\frac{T_f}{T_o}\right) - R \ln\left(\frac{P_f}{P_o}\right) && (11.24)\\ &= C_p \ln\left(\frac{v_f}{v_o}\right) + C_v \ln\left(\frac{P_f}{P_o}\right) && (11.25)\end{aligned}$$

- For reversible flow through a compressor, pump, fan or blower

$$(\dot{W})_{in} = \dot{m} \int_{P_i}^{P_e} v \, dP \qquad (11.37)$$

- For reversible liquid flow through a pump

$$(\dot{W})_{in} \approx \dot{m} \, v_i \, (P_e - P_i) \qquad (11.38)$$

since the specific volume of a liquid varies only slightly when it flows through a pump.