Chapter 8

ALL ABOUT HEAT

> Hot water and cold water "contain" the same amount of heat: none at all.
>
> – J.P. Holman (*Thermodynamics*)

A beginner in thermodynamics may think that a hot object contains more heat than what a cold object contains. The truth is none of them contains any heat at all since heat does not reside in an object. The beginner may also think that the temperature of an object cannot be raised unless heat is provided to the object, which is a wrong notion. Even one with a fair knowledge of thermodynamics, often confuses heat with enthalpy. This chapter is presented to give a clear idea about what heat is, and what heat is not.

8.1 What is Heat?

We know heat is energy. In the past, even scientists and engineers did not know that. They thought heat was a fluid. Almost 50 years after the construction of the first successful steam engine in 1712 by Thomas Newcomen, Professor Joseph Black founded a theory on heat. This theory is known as caloric theory, and it said heat was a colourless, weightless fluid known as caloric, and it was conserved. That means caloric, or heat, could not be created or destroyed, but it could transfer itself from one object to another. If a metal was heated using fire, it was explained, then the fluid caloric was transferred from the fire to the metal. James Watt, associated with Professor Black's laboratory, modified Thomas Newcomen's engine in 1765 and made the first efficient steam engine. In a steam engine, heat generated from burning the coal is converted into work required to do a job, such as rotating the wheels of a train.

In 1824, a French engineer named Sadi Carnot, a believer in the caloric theory, presented on paper an ideal engine that could provide the maximum amount of work for a specific amount of heat given to the engine. Carnot theorized that the work was obtained from an engine because of heat, which he believed as fluid, falling from a high temperature source to a low temperature source. He showed that the thermal efficiency of his ideal engine depended only on these two temperatures. No real engine can be more efficient than the Carnot engine, and this result, very interestingly, is still valid, even though Carnot believed that heat was fluid.

James Prescott Joule, in 1840s, performed a series of experiments where falling wights stirred a liquid and heated it up. He showed that the heat produced had always the same quantitative relationship to the energy lost by the falling weights, and concluded that heat was just another form of energy. Joule's ideas and those of Carnot were reconciled, simply and effectively, by Rudolf Julius Emanuel Clausius in 1850 who wrote down the First law of Thermodynamics as "the total energy of the system is a constant". William John Macquorn Rankine, a Scottish engineer, defined thermodynamic efficiency of a heat engine in 1853 when applying the theory of thermodynamics to heat engines, and wrote the first thermodynamics textbook in 1859.

Our world is full of heat engines where heat is converted into useful work. Car engines and jet engines, for example, are powered by the heat generated from burning a fuel. The source of electricity generation in coal, thermal and nuclear power stations is heat. Heat released from the burning of coal, petroleum and natural gas accounts for about 85% of the global energy consumption, which is in the order of hundreds of exajoules.

Without the recognition of the exact nature of heat and its relationship to work, the world would not have gone so far in its utilization of heat to provide for the gigantic amount of energy consumed by the human race today. This colossal amount of energy consumption has resulted in pollution of all kinds. One of the consequences of which is global warming, and the resulting life-threatening climate change.

8.2 Heat Supply and Common Sense

We know that heat flows from a hot object to a cold object when the two touch each other. Thus, we have a tendency to believe that a hot object contains more heat than a cold object does. The truth is heat does not reside in an object, and there is no such thing as the heat content of an object. The thermal energy that resides in an object is not heat, but internal energy. When a hot object comes into contact with a cold object, the internal energy content of the hot object decreases and the internal energy content of the cold object increases, until the temperatures of both become the same. The energy that is transferred between the two objects during such a process, driven by the temperature gradient between the two objects, is called heat. This means, we can refer to a form of energy as heat only when it is being transferred from one object to the other because of the temperature difference existing between the two objects.

When heat is supplied to water, the temperature of water increases. Such familiar everyday experiences have made some of us to conclude that 'when heat is provided to a substance, its temperature should necessarily increase'. Is that really true? If so, how could we explain the following observations?

- Heat provided to ice at its melting point turns ice to water at the temperature of the melting point. That is, the temperature remains constant but there is change of phase.
- When heat is added to water at its boiling temperature, the temperature does not change until all water is turned into steam. That is, there is phase change at constant temperature.

Now, some may conclude that 'when a substance is heated either the temperature of the substance should increase or the phase of the substance should change'. Is that true? No. The following sections are designed to give us an insight into what else may happen when heat is supplied to a substance.

8.3 Heat Supplied to Increase the Temperature

Take air in a cylindrical container with enough force applied to the piston to keep the air volume constant, as in Figure 8.1. Insulate the walls of the cylinder and the piston so that heat is not lost to the surroundings. Supply heat to air using a heating coil.

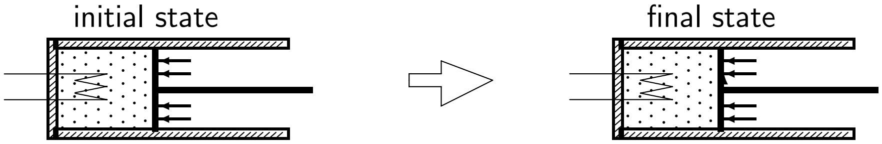

Figure 8.1 Heat supplied at constant volume.

Since no work transfer occurs, the heat supplied to air goes to increase the internal energy of air in accordance with the first law of thermodynamics

$$Q_{in} = \Delta U$$

Assuming ideal gas behaviour, the increase in the specific internal energy of air may be accounted for by

$$\Delta u = \int_{T_o}^{T_f} C_v \, dT$$

Combining the two equations above, we get

$$Q_{in} = m \int_{T_o}^{T_f} C_v \, dT$$

which says that the heat supplied to a substance increases its temperature. It is a very familiar experience for us, and therefore we have no problem recognizing it, and accepting it.

8.4 Heat Supplied to Do Work

Consider air in the set up shown in Figure 8.2, and let us carry out the following thought experiment. Allow the piston to move away by a tiny distance such that the volume occupied by the air in the cylinder is increased by a tiny amount. Do not supply heat to the air during this step. The tiny increase in the volume of air would cause a tiny decrease in the air pressure and hence the temperature of air would drop by a tiny amount. Let us now hold the volume of the air in the cylinder constant at its new value, and supply heat to the air such that the temperature of air is brought back to its original value.

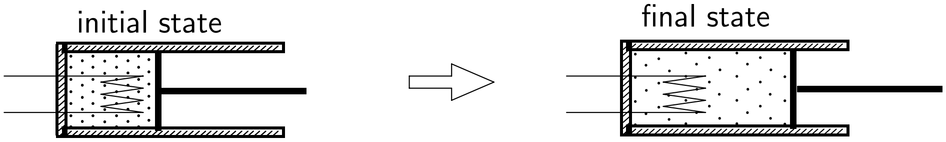

Figure 8.2 Heat supplied maintaining temperature constant.

By continuing this procedure, it is possible to supply heat to the air while maintaining its temperature about a constant value. Here we have a process in which the heat supplied neither increases the temperature nor changes the phase of a substance. We may ask then what happened to the heat supplied to air. The answer to the question shall be found as follows.

First law of thermodynamics applied to the air assumed to behave as an ideal gas, gives

$$Q_{in} = -W_{in} + \Delta U = W_{out} + m \int_{T_o}^{T_f} C_v \, dT$$

Since T is maintained a constant, the above becomes $Q_{in} = W_{out}$. That is, all the heat supplied to the ideal gas maintained at a constant temperature is used by the ideal gas to do work in pushing the piston away.

8.5 Temperature Increase without Heat Supply

Consider air in the set up shown in Figure 8.3. Insulate the system so that heat does not cross the system boundary. Push the piston slowly to compress the air. Since no heat is transferred across the system boundary, the first law of thermodynamics applied to air yields

$$W_{in} = \Delta U = m \int_{T_o}^{T_f} C_v \, dT$$

if ideal gas behaviour is assumed. The above expression clearly shows that the work done on the air is responsible for increasing the temperature. That is, the temperature of air increases with no heat supplied to it.

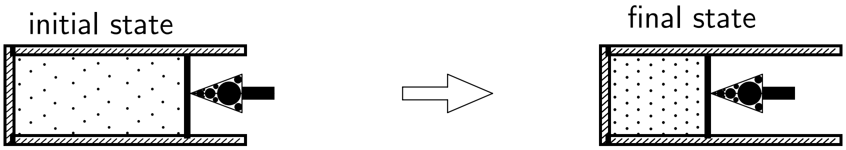

Figure 8.3 No heat is supplied, but force is applied on the piston.

Summarizing what we have learnt so far in this chapter, we could say that changes in the properties, such as the temperature, pressure or volume of a system, may alter the internal energy of the system. Remember that temperature, pressure, volume and internal energy are all properties of a system, and are related to each other. Any change in the internal energy of a closed system, in turn, is related to heat and/or work transfers between the system and its surroundings by the first law of thermodynamics applied to a closed system, which is

$$Q_{in} + W_{in} = \Delta U$$

Let us once again state that heat is defined as the form in which energy crosses the boundary of a system owing to a temperature difference between the system and its surroundings. Any other form of energy transfer across the boundary of a system is in the form of work, such as the boundary work.

8.6 Direct Evaluation of Q_{in}

In the examples that we have so far worked out, the system was either taken as an adiabatic system, for which $Q_{in} = 0$, or the numerical value of Q_{in} or Q_{out} was given. Whenever we were asked to determine how much heat was transferred to or from the system, we calculated the value of Q_{in} by use of the first law of thermodynamics. In this section, we will explore how else Q_{in} could be evaluated.

Suppose a system is heated by a heat source, such as an electric heater. The amount of heat added to the system from the heat source can be evaluated using

$$Q_{in} = \int_{t_o}^{t_f} \dot{\lambda}\, \delta t$$

where $\dot{\lambda}$ is the rate at which heat is added to the system from the heat source, and t_o and t_f are the initial and final times, respectively.

If $\dot{\lambda}$ is a constant then we have

$$Q_{in} = \dot{\lambda}(t_f - t_o)$$

where, if $\dot{\lambda}$ is in kW and t_f and t_o are in in seconds, Q_{in} will be in kJ.

Consider a system heated by heat transferred to it from its surroundings. We know that heat flows into or out of a system, only if there is a temperature difference between the system and surroundings. Let us say that the system is at temperature T_{sys}, and the surroundings is at temperature T_{surr}, which is higher than T_{sys}. The heat transfer dQ_{in} during a very short time dt to the system from the surroundings is then given by

$$dQ_{in} = \kappa\, A\, (T_{surr} - T_{sys})\, dt$$

where κ is the overall heat transfer coefficient in kJ/m$^2\cdot$K$\cdot$s, and A is the surface area in m^2 across which heat enters the system from the surroundings.

If the surroundings are at a lower temperature than the system, then T_{surr} is less than T_{sys}. Consequently, dQ_{in} of the above expression takes a negative value. It means that the heat flows from the system to its surroundings.

To determine the total heat transfer, the above equation is integrated to obtain

$$Q_{in} = \int_{t_o}^{t_f} \kappa\, A\, (T_{surr} - T_{sys})\, dt$$

To evaluate the above integral we need to know how the temperature of the system and the temperature of its surroundings vary with time.

Owing to the complex nature of the evaluation of Q_{in} using the above expression, beginners in thermodynamics are seldom expected to evaluate Q_{in} using the above described method.

8.7 Zeroth Law of Thermodynamics

Let us now look at something that is seemingly obvious to us. We know that heat flows from a hot object to a cold object when they are brought into contact. If no heat flows between them when they are brought into contact, then the two objects should be at the same temperature. These two objects are then in thermal equilibrium with each other.

The zeroth law of thermodynamics, formulated in 1931, states that if a system A is in thermal equilibrium with a system C and another system

B is also in thermal equilibrium with the system C, then the systems A and B will be in thermal equilibrium with each other.

Let us consider systems A and B as liquids in two different containers which are not in contact, and system C as a thermometer. Let us suppose that the two systems A and B are in thermal equilibrium with the thermometer C, which means that systems A and B give the same temperature reading. According to the zeroth law, the two systems A and B are in thermal equilibrium even if they are not in contact since they have the same temperature reading.

The zeroth law, like the first law of thermodynamics, is not provable, even though it seems obvious and trivial.

8.8 Heat and Enthalpy

Even one with a fair knowledge of thermodynamics often confuses heat with enthalpy. This section would help us to see clearly the relationship between heat and enthalpy.

Enthalpy defined by (4.1) as $H = U + PV$, when differentiated, takes the form

$$dH = dU + P\,dV + V\,dP \tag{8.1}$$

Substituting dU from (3.5), which is the differential form of the first law applied to closed simple compressible systems, in (8.1), we get

$$dH = dQ_{in} + dW_{in} + P\,dV + V\,dP$$

which could be rearranged to yield

$$dQ_{in} = dH - dW_{in} - P\,dV - V\,dP \tag{8.2}$$

On integration of (8.2), we get

$$Q_{in} = \Delta H - W_{in} - \int_{V_o}^{V_f} P\,dV - \int_{P_o}^{P_f} V\,dP \tag{8.3}$$

which relates the heat provided to a closed simple compressible system to the enthalpy increase of the system. Remembering (8.3) would keep us away from confusing heat with enthalpy.

If the closed system undergoes a quasistatic process then dW_{in} given by (7.5) will be used to reduce (8.2) to

$$dQ_{in} = dH - V\,dP \tag{8.4}$$

If the given process is a constant-pressure process then $dP = 0$. Thus, (8.4) becomes

$$dQ_{in} = dH \tag{8.5}$$

which upon integration yields

$$Q_{in} = \Delta H \tag{8.6}$$

Equation (8.6) is applicable to a **a closed system undergoing a quasistatic constant-pressure process involving no forms of work transfer other than boundary work.**

It is important to note that enthalpy is a property of a system, and therefore the enthalpy change ΔH is the difference between the enthalpies at the initial and the final states. The amount of heat entering the system Q_{in} depends on the path that the system takes between the initial and the final states of the system. And, these two very different entities equal each other only under special circumstances such as the one above.

8.9 Heat and Internal Energy

The circumstances under which heat exchanged with a system could be related to the internal energy change of a system is explored in this section. Let us consider a closed system undergoing a constant-volume process. The first law, given by (3.4), applied to this process becomes

$$Q_{in} = \Delta U \tag{8.7}$$

provided there is no other forms of work transfer associated with the closed system.

It is important to note that the internal energy change ΔU is the difference between the internal energies at the initial and the final states. The amount of heat entering the system Q_{in} depends on the path that the system takes between the initial and the final states of the system.

And, these two very different entities equal each other only under special circumstances such as the one above.

8.10 Heat and Specific Heat

We have learned about the specific heat at constant volume (C_v) in Section 5.4, and about the specific heat at constant pressure (C_p) in Section 5.5. Nowhere in these sections, however, we find any reference to the quantity heat. Then, why do we call C_v and C_p specific heats? It is a question asked by many beginners in thermodynamics. In this section, we shall see how to relate the specific heats to the quantity heat.

First, let us deal with specific heat at constant pressure. From the expression for C_p given by (5.9), we get

$$C_p = \left.\frac{dh}{dT}\right|_P = \left.\frac{1}{m}\frac{dH}{dT}\right|_P \tag{8.8}$$

When using (8.5) applicable to a quasistatic constant-pressure process involving only boundary work, (8.8) becomes

$$C_p \equiv \left.\frac{1}{m}\frac{dQ_{in}}{dT}\right|_P \tag{8.9}$$

Therefore, C_p is the heat required to raise the temperature of a unit mass of a substance by one degree in a quasistatic constant-pressure process involving no forms of work transfer other than boundary work.

From the expression for C_v given by (5.5), we get

$$C_v = \left.\frac{du}{dT}\right|_v = \left.\frac{1}{m}\frac{dU}{dT}\right|_v \tag{8.10}$$

For a constant-volume process, the boundary work is zero, and therefore the first law yields $dQ_{in} = dU$ provided no other forms of work transfer is involved. Thus, (8.10) becomes

$$C_v \equiv \left.\frac{1}{m}\frac{dQ_{in}}{dT}\right|_v \tag{8.11}$$

Therefore, C_v is the heat required to raise the temperature of a unit mass of a substance by one degree in a constant-volume process involving no forms of work transfer.

Note that some textbooks introduce C_p and C_v in terms of (8.9) and (8.11), respectively. Upon integration of (8.9) and (8.11), we get the following two very useful expressions for the direct evaluation of heat in the following processes:

For a quasistatic constant-pressure process involving no forms of work transfer other than boundary work:

$$Q_{in} = m \int_{T_o}^{T_f} C_p \, dT \tag{8.12}$$

For a constant-volume process involving no forms of work transfer:

$$Q_{in} = m \int_{T_o}^{T_f} C_v \, dT \tag{8.13}$$

Note that (8.12) and (8.13) are applicable for any simple compressible substance.

8.11 Worked Examples

Example 8.1

An ideal gas ($C_v = 0.744$ kJ/kg · K) in a piston-cylinder arrangement is compressed such that 93 kJ/kg of work is done on the gas. Assuming adiabatic condition prevails, find out what happens to the work provided to the system.

Solution to Example 8.1

The first law of thermodynamics applied to the given adiabatic system gives $W_{in} = \Delta U = m\,C_v\,\Delta T$. Therefore,

$$\Delta T = \frac{W_{in}}{m\,C_v} = \frac{93}{0.744} = 125 \text{ K}$$

That is, the work provided to the system has increased the temperature of the system. Note that the temperature of the system has increased without receiving any heat.

Example 8.2

The piston-cylinder device in Figure 8.4 contains 0.2 kg of air with a molecular weight of 29, C_v of 0.718 kJ/kg · K and C_p of 1.005 kJ/kg · K. The initial pressure of air is 1 MPa, and it is just enough to balance the weight of the piston and the atmospheric pressure acting on the piston. The initial temperature is 127°C. Heat is transferred to air until the piston, assumed to be frictionless, reaches the stops. Further heat is transferred to air until the air reaches 2 MPa and 927°C. Sketch the path of the process on a P-V diagram, and determine the amount of heat supplied to the air.

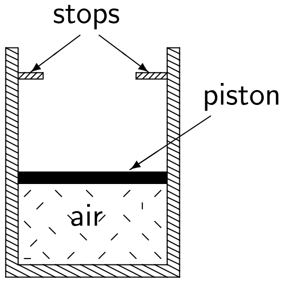

Figure 8.4

Solution to Example 8.2

At the initial state A, $P_A = 1$ MPa and $T_A = 400$ K. Heat is transferred to air so that it expands until the piston reaches the stops shown in Figure 8.4. This intermediate state is denoted by B. During process A→B, the expansion of air is at a constant pressure of 1 MPa, since this pressure just balances the weight of the piston and the atmospheric pressure acting on the piston. Thus $P_B = P_A = 1$ MPa. Assume this constant-pressure process to be quasistatic. After the piston reaches the stops, further heat is added to the air until the process reaches its final state, denoted by C, at which $P_C = 2$ MPa and $T_C = 1200$ K. During process B→C, the volume of air is a constant. The path A→B→C on the P-V diagram of Figure 8.5 describes the entire process.

To evaluate the total amount of heat transferred to air during the entire process, let us determine the heat transfers separately for the processes A→B and B→C. Since A→B is a quasistatic constant-pressure process, we can use (8.12) to determine the heat transfer as

$$(Q_{in})_{A\to B} = m\,C_p\,(T_B - T_A) \tag{8.14}$$

Since B→C is a constant-volume process of a simple compressible closed system, we can use (8.13) to determine the heat transfer as

$$(Q_{in})_{B\to C} = m\,C_v\,(T_C - T_B) \tag{8.15}$$

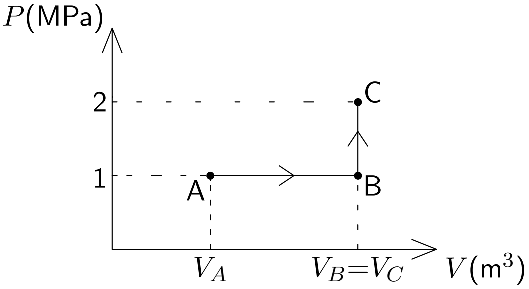

Figure 8.5 The path of the process given in Example 8.2.

Adding (8.14) and (8.15), we get

$$(Q_{in})_{A\to B\to C} = m\,C_p\,(T_B - T_A) + m\,C_v\,(T_C - T_B)$$

Substituting all the numerical values that we already know, we get

$$(Q_{in})_{A\to B\to C} = 0.2\times 1.005\times(T_B - 400) + 0.2\times 0.718\times(1200 - T_B) \tag{8.16}$$

where the unknown T_B can be found using

$$T_B = \frac{P_B\,T_C}{P_C} = \frac{1\times 1200\text{ K}}{2} = 600\text{ K},$$

at states B and C along the constant-volume path B→C. Using the numerical value of T_B in (8.16), we get $(Q_{in})_{A\to B\to C}$ = 126.4 kJ.

Example 8.3

An ideal gas of 0.01 kmol is taken through a cyclic process consisting of the following four processes: Process A to B is an isothermal expansion at 800 K from 8 bar to 6 bar; Process B to C is an adiabatic expansion to 3 bar; Process C to D is a constant-pressure cooling; Process D to A is a constant-volume heating. Sketch the cyclic process on a P-V diagram. Assuming that all processes are quasistatic and taking γ to be 1.38, determine the temperatures at states C and D, and calculate the heat and work transfers for the entire cyclic process.

Solution to Example 8.3

The cyclic process sketched on a P-V diagram is shown in Figure 8.6.

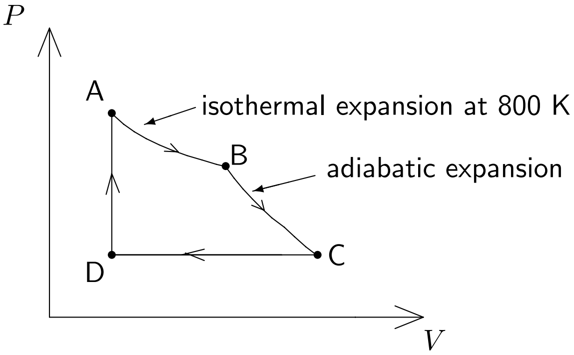

Figure 8.6 The path of the cyclic process of Example 8.3.

(a) Determination of the temperatures at states C and D:

Table 8.1 shows the data at states A, B, C and D. Since B → C is a quasistatic adiabatic expansion of an ideal gas, (7.31) can be used to find T_C as

$$T_C = \left(\frac{P_C}{P_B}\right)^{(\gamma-1)/\gamma} T_B = \left(\frac{3}{6}\right)^{(1.38-1)/1.38} \times 800 \text{ K} = 661 \text{ K}$$

Since D → A is a constant-volume process of an ideal gas, the ideal gas equation of state can be used to find T_D as

$$T_D = \left(\frac{P_D}{P_A}\right) T_A = \left(\frac{3}{8}\right) \times 800 \text{ K} = 300 \text{ K}$$

A	$P_A = 8$ bar	$T_A = 800$ K	$V_A = ?$
B	$P_B = 6$ bar	$T_B = 800$ K	$V_B = ?$
C	$P_C = 3$ bar	$T_C = ?$	$V_C = ?$
D	$P_D = 3$ bar	$T_D = ?$	$V_D = V_A = ?$

Table 8.1 Data at states A, B, C and D of the cyclic process.

(b) Calculation of the heat and work transfers:

Process A → B is a quasistatic isothermal expansion of an ideal gas at 800 K, and therefore the work transfer is calculated using

$$\begin{aligned}(W_{in})_{A\to B} &= -\int_{V_A}^{V_B} P\,dV = -n\,R\,T_A\,\ln\left(\frac{P_A}{P_B}\right)\\ &= -0.01\times 8.314\times 800\times\ln\left(\frac{8}{6}\right)\text{ kJ} = -19.1\text{ kJ}\end{aligned}$$

and the heat transfer is calculated using

$$(Q_{in})_{A\to B} = -(W_{in})_{A\to B} = 19.1\text{ kJ},$$

since there is no internal energy change for an isothermal process of an ideal gas.

Process B → C is a quasistatic adiabatic expansion of an ideal gas, and therefore the heat transfer becomes $(Q_{in})_{B\to C} = 0$, and the work transfer is calculated using

$$\begin{aligned}(W_{in})_{B\to C} &= (\Delta U)_{B\to C} = n\,C_v\,(T_C - T_B) = n\,\frac{R}{\gamma-1}\,(T_C - T_B)\\ &= 0.01\times\frac{8.314}{1.38-1}\times(661-800)\text{ kJ} = -30.4\text{ kJ}\end{aligned}$$

Process C → D is a quasistatic constant-pressure cooling of an ideal gas, and therefore the heat transfer is calculated using (8.12) as

$$\begin{aligned}(Q_{in})_{C\to D} &= n\,C_p\,(T_D - T_C) = n\,\frac{\gamma\,R}{\gamma-1}\,(T_D - T_C)\\ &= 0.01\times\frac{1.38\times 8.314}{1.38-1}\times(300-661)\text{ kJ} = -109.0\text{ kJ}\end{aligned}$$

and the work transfer is calculated using

$$\begin{aligned}(W_{in})_{C\to D} &= (\Delta U)_{C\to D} - (Q_{in})_{C\to D}\\ &= = n\,C_v\,(T_D - T_C) - n\,C_p\,(T_D - T_C) = -n\,R\,(T_D - T_C)\\ &= -0.01\times 8.314\times(300-661)\text{ kJ} = 30.0\text{ kJ}\end{aligned}$$

Process D → A is a constant-volume heating of an ideal gas, and therefore the work transfer becomes $(W_{in})_{D\to A} = 0$, and the heat transfer is calculated using (8.13) as

$$\begin{aligned}(Q_{in})_{D\to A} &= = n\,C_v\,(T_A - T_D) = n\,\frac{R}{\gamma-1}\,(T_A - T_D)\\ &= 0.01\times\frac{8.314}{1.38-1}\times(800-300)\text{ kJ} = 109.4\text{ kJ}\end{aligned}$$

The net heat transfer for the entire cyclic process therefore becomes

$$(Q_{in})_{net} = (19.1 + 0 - 109.0 + 109.4)\ \text{kJ} = 19.5\ \text{kJ}$$

and the net work transfer for the entire cyclic process becomes

$$(W_{in})_{net} = (-19.1 - 30.4 + 30.0 + 0)\ \text{kJ} = -19.5\ \text{kJ}$$

Comment: Note that we have $(Q_{in})_{net} + (W_{in})_{net} = 0$. It is because in a cyclic process the net internal energy change is zero owing to the initial and the final states being the same.

Example 8.4

An ideal gas enclosed in a piston-cylinder assembly is compressed adiabatically to increase its temperature from T_L K to T_H K. Heat is then supplied to the ideal gas such that it expands isothermally at T_H K. The heat supply is cut off and the ideal gas is allowed to continue expanding adiabatically until its temperature drops to T_L K. Finally, the ideal gas is compressed isothermally until it returns to its initial state during which heat is rejected to the surroundings. Sketch the cyclic process on a P-V diagram. Assuming that all processes are quasistatic, obtain an expression, in terms of T_L and T_H, for the thermal efficiency of the cycle, η_{th}, defined as the net work output of the cycle per unit of heat added to the cycle.

If $T_L = 300$ K and $T_H = 1500$ K, determine the amount of heat added to the ideal gas to produce 100 kJ of net work output. Also, determine the amount of heat rejected to the surroundings by the ideal gas.

Solution to Example 8.4

The cyclic process sketched on a P-V diagram is shown in Figure 8.7, where A → B is the quasistatic adiabatic compression, B → C is the quasistatic isothermal heating at the temperature T_H, C → D is the quasistatic adiabatic expansion, and D → A is the quasistatic isothermal cooling at the temperature T_L.

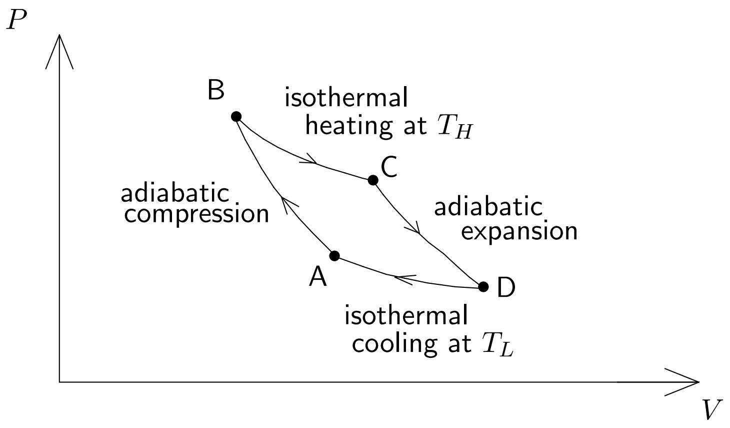

Figure 8.7 The path of the cyclic process of Example 8.4.

The thermal efficiency of the cycle, η_{th}, is defined as the net work output of the cycle, W_{net}, per unit of heat added to the cycle, Q_{in}, and therefore we have

$$\eta_{th} \equiv \frac{W_{net}}{Q_{in}} \tag{8.17}$$

In a cyclic process, the net change in the internal energy is zero, and therefore the first law applied to the cyclic process gives

$$W_{net} = Q_{in} - Q_{out} \tag{8.18}$$

where Q_{in} is the heat added to the ideal gas during the isothermal heating B → C and Q_{out} is the heat removed from the ideal gas during the isothermal cooling D → A. No heat is added or removed during the adiabatic processes A → B and C → D.

Combining (8.17) and (8.18), we get

$$\eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \tag{8.19}$$

Since the internal energy remains constant for an isothermal process of an ideal gas, isothermal heat addition during B → C may be expressed using the first law as

$$Q_{in} = -W_{in} = n \int_{V_B}^{V_C} P\,dV = n\,R\,T_H \ln\left(\frac{V_C}{V_B}\right) \tag{8.20}$$

where T_H in the temperature of the ideal gas during the isothermal heating process. Isothermal heat rejection during D → A may be expressed using the first law as

$$Q_{out} = W_{in} = -n \int_{V_D}^{V_A} P\, dV = -n\, R T_L \ln\left(\frac{V_A}{V_D}\right) = n\, R T_L \ln\left(\frac{V_D}{V_A}\right) \tag{8.21}$$

where T_L in the temperature of the ideal gas during the isothermal cooling process.

Combining (8.19), (8.20) and (8.21), we have

$$\eta_{th} = 1 - \frac{T_L \ln(V_D/V_A)}{T_H \ln(V_C/V_B)} \tag{8.22}$$

Since A → B is a quasistatic adiabatic expansion of an ideal gas, (7.30) can be used to get

$$\frac{T_A}{T_B} = \left(\frac{V_B}{V_A}\right)^{\gamma-1} \tag{8.23}$$

Since C → D is a quasistatic adiabatic expansion of an ideal gas, (7.30) can be used to get

$$\frac{T_C}{T_D} = \left(\frac{V_D}{V_C}\right)^{\gamma-1} \tag{8.24}$$

Combining (8.23) and (8.24) using $T_B = T_C = T_H$ and $T_D = T_A = T_L$, we have

$$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

which can be rearranged to give

$$\frac{V_D}{V_A} = \frac{V_C}{V_B} \tag{8.25}$$

Using (8.25), we can reduce (8.22) to

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

The above expression for thermal efficiency is known as the Carnot efficiency, and therefore we write it as follows:

$$\eta_{Carnot} \equiv 1 - \frac{T_L}{T_H} \tag{8.26}$$

If T_L = 300 K and T_H = 1500 K, then (8.26) gives

$$\eta_{Carnot} = 1 - \frac{300}{1500} = 0.80 = 80\%$$

Since the thermal efficiency of the cycle is 80%, using (8.17), we can calculate the amount of heat to be added to the cycle to produce 100 kJ of net work output from the cycle as,

$$Q_{in} = \frac{W_{net}}{\eta_{Carnot}} = \frac{100 \text{ kJ}}{0.8} = 125 \text{ kJ}$$

The amount of heat rejected to the surroundings from the cycle can be calculated using (8.18) as

$$Q_{out} = Q_{in} - W_{net} = 125 \text{ kJ} - 100 \text{ kJ} = 25 \text{ kJ}$$

Comment: Carnot efficiency is a very important concept in thermodynamics. It is because the thermal efficiency of no engine, that converts heat to work operating in a cyclic process between the maximum temperature T_H K and the minimum temperature T_L K, can be higher than the Carnot efficiency (the proof of which is given in Chapter 13), which Sadi Carnot presented in 1824, while not knowing for sure that heat is energy.

Example 8.5

In a cyclic process, air initially at 1 bar and 300 K is compressed adiabatically to reduce its volume to one eighth of the initial value. The compressed air is heated at constant volume to 1500 K. The air in then expanded adiabatically and finally cooled at constant volume to its initial state. Sketch the cyclic process on a P-V diagram. Assume that all processes are quasistatic and that air behaves as as ideal gas with γ =1.4. Determine the thermal efficiency of the cycle and the amount of heat added to the air to produce 100 kJ of net work output.

Determine the numerical values of the pressures at the end of each process in the cycle.

Solution to Example 8.5

The cyclic process sketched on a P-V diagram is shown in Figure 8.8, where A → B is the quasistatic adiabatic compression, B → C is the constant-volume heating, C → D is the quasistatic adiabatic expansion, and D → A is the constant-volume cooling.

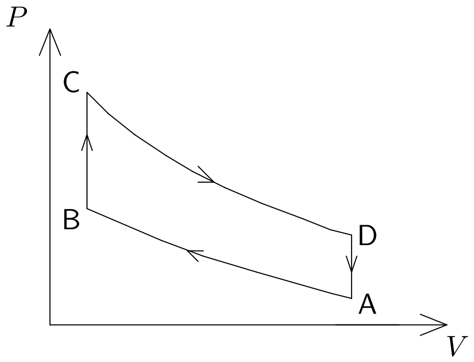

Figure 8.8 The path of the cyclic process of Example 8.5.

Let us first determine the thermal efficiency of the cycle, η_{th}, the definition of which is given by (8.17). For the given cyclic process, we can calculate η_{th} using (8.19), where Q_{in} is the amount of heat added during the constant-volume heating B → C, and Q_{out} is the amount of heat rejected during the constant-volume cooling D → A. No heat is added or removed during the adiabatic processes A → B and C → D.

Constant-volume heat addition can be calculated using (8.13) as

$$Q_{in} = m\,C_v\,(T_C - T_B) \tag{8.27}$$

Constant-volume heat rejection can be calculated using (8.13) as

$$Q_{out} = -m\,C_v\,(T_A - T_D) = m\,C_v\,(T_D - T_A) \tag{8.28}$$

Combining (8.19), (8.27) and (8.28), we have

$$\eta_{th} = 1 - \frac{m\,C_v\,(T_D - T_A)}{m\,C_v\,(T_C - T_B)} = 1 - \frac{T_D - T_A}{T_C - T_B} \tag{8.29}$$

where $T_A = 300$ K, $T_C = 1500$ K and T_B and T_D are unknown. To determine T_B and T_D, let us tabulate all the known data at the states A, B, C and D as in Table 8.2.

A	$P_A = 1$ bar	$T_A = 300$ K	$V_A = ?$
B	$P_B = ?$	$T_B = ?$	$V_B = V_A/8$
C	$P_C = ?$	$T_C = 1500$ K	$V_C = V_B = V_A/8$
D	$P_D = ?$	$T_D = ?$	$V_D = V_A = ?$

Table 8.2 Data at states A, B, C and D of the cyclic process.

Since A → B is a quasistatic adiabatic compression of an ideal gas, (7.30) can be used to find T_B as

$$T_B = \left(\frac{V_A}{V_B}\right)^{(\gamma-1)} T_A = 8^{0.4} \times 300 \text{ K} = 689 \text{ K}$$

Since C → D is a quasistatic adiabatic expansion of an ideal gas, (7.30) can be used to find T_D as

$$T_D = \left(\frac{V_C}{V_D}\right)^{(\gamma-1)} T_C = \left(\frac{1}{8}\right)^{0.4} \times 1500 \text{ K} = 653 \text{ K}$$

Substituting the numerical values of the temperatures in (8.29), we get

$$\eta_{th} = 1 - \frac{653 - 300}{1500 - 689} = 0.565 = 56.5\%$$

Since the thermal efficiency of the cycle is 56.5%, using (8.17), we can calculate the amount of heat to be added to the cycle to produce 100 kJ of net work output from the cycle as,

$$Q_{in} = \frac{W_{net}}{\eta_{th}} = \frac{100 \text{ kJ}}{0.565} = 177 \text{ kJ}$$

The amount of heat rejected to the surroundings from the cycle can be calculated using (8.18) as

$$Q_{out} = Q_{in} - W_{net} = 177 \text{ kJ} - 100 \text{ kJ} = 77 \text{ kJ}$$

We know $P_A = 1$ bar, and we are to find P_B, P_C and P_D. Since A → B is a quasistatic adiabatic compression of an ideal gas, (7.29) can be used to find P_B as

$$P_B = \left(\frac{V_A}{V_B}\right)^{\gamma} P_A = 8^{1.4} \times 1 \text{ bar} = 18.4 \text{ bar}$$

Since B → C is a constant-volume process of an ideal gas, ideal gas equation of state can be used to find P_B as

$$P_C = \left(\frac{T_C}{T_B}\right) P_B = \frac{1500}{689} \times 18.4 \text{ bar} = 40.1 \text{ bar}$$

Since C → D is a quasistatic adiabatic expansion of an ideal gas, (7.29) can be used to find P_D as

$$P_D = \left(\frac{V_C}{V_D}\right)^{\gamma} P_C = \left(\frac{1}{8}\right)^{1.4} \times 40.1 \text{ bar} = 2.2 \text{ bar}$$

Comment: The cycle studied in this problem is known as the ideal Otto cycle. It is an idealized cycle used to understand the working of an engine in which the fuel-air mixture contained in a piston-cylinder arrangement is first compressed and then ignited using a spark, as in a 4-stroke car engine fueled by petrol. Also, notice that the efficiency of the ideal Otto cycle operating between 1500 K and 300 K is only 56.5%, where as the Carnot efficiency for the same temperature extremes is 80% (see **Example 8.4**).

Example 8.6

In a cyclic process, air initially at 1 bar and 300 K is compressed adiabatically to reduce its volume to one twentieth of the initial value. The compressed air is heated at constant pressure to 1500 K. The air in then expanded adiabatically and finally cooled at constant volume to its initial state. Sketch the cyclic process on a P-V diagram. Assuming that all processes are quasistatic and that air behaves as as ideal gas with γ =1.4, determine the thermal efficiency of the cycle, η_{th}.

Determine the quantity of heat added and the net work output per kg of air, taking the molar mass of the air as 29 kg/kmol.

Solution to Example 8.6

The cyclic process sketched on a P-V diagram is shown in Figure 8.9.

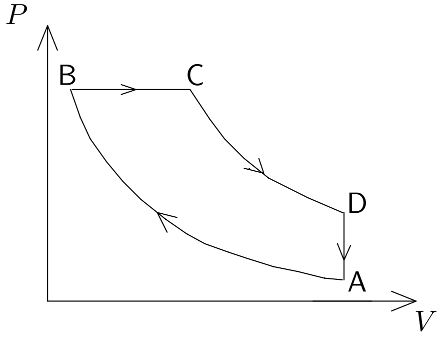

Figure 8.9 The path of the cyclic process of Example 8.6.

A → B is the quasistatic adiabatic compression, B → C is the constant-pressure heating, C → D is the quasistatic adiabatic expansion, and D → A is the constant-volume cooling.

The thermal efficiency of the cycle, η_{th}, is defined by (8.17). For the given cyclic process, we can calculate η_{th} using (8.19), where Q_{in} is the amount of heat added during the constant-pressure heating B → C, and Q_{out} is the amount of heat rejected during the constant-pressure cooling D → A. No heat is added or removed during the adiabatic processes A → B and C → D. Constant-pressure heat addition can be calculated using (8.12) as

$$Q_{in} = m\,C_p\,(T_C - T_B) \tag{8.30}$$

Constant-volume heat removal can be calculated using (8.13) as

$$Q_{out} = -m\,C_v\,(T_A - T_D) = m\,C_v\,(T_D - T_A) \tag{8.31}$$

Combining (8.19), (8.30) and (8.31), we have

$$\eta_{th} = 1 - \frac{1}{\gamma}\left(\frac{T_D - T_A}{T_C - T_B}\right) \tag{8.32}$$

where $T_A = 300$ K, $T_C = 1500$ K and T_B and T_D are unknown. To determine T_B and T_D, let us tabulate all the known data at the states A, B, C and D as in Table 8.3.

A	$P_A = 1$ bar	$T_A = 300$ K	$V_A = ?$
B	$P_B = ?$	$T_B = ?$	$V_B = V_A/20$
C	$P_C = P_B = ?$	$T_C = 1500$ K	V_C
D	$P_D = ?$	$T_D = ?$	$V_D = V_A = ?$

Table 8.3 Data at states A, B, C and D of the cyclic process.

Since A → B is a quasistatic adiabatic compression of an ideal gas, (7.30) can be used to find T_B as

$$T_B = \left(\frac{V_A}{V_B}\right)^{(\gamma-1)} T_A = 20^{0.4} \times 300\text{ K} = 994\text{ K} \tag{8.33}$$

Since C → D is a quasistatic adiabatic expansion of an ideal gas, (7.30) can be used to find T_D as

$$T_D = \left(\frac{V_C}{V_D}\right)^{(\gamma-1)} T_C = \left(\frac{V_C}{V_A}\right)^{0.4} \times 1500\text{ K} \tag{8.34}$$

where the unknown V_C/V_A can be found, using the fact that B → C is a constant-pressure expansion of an ideal gas, as follows:

$$\begin{aligned}\frac{V_C}{T_C} &= \frac{V_B}{T_B}\\ \frac{V_C}{V_B} &= \frac{T_C}{T_B}\\ \frac{V_C}{V_A/20} &= \frac{T_C}{T_B}\\ \frac{V_C}{V_A} &= \frac{T_C}{20 \times T_B} = \frac{1500}{20 \times 994} = 0.076 \qquad (8.35)\end{aligned}$$

Combining (8.34) and (8.35), we get

$$T_D = 0.076^{0.4}\,(1500\ \text{K}) = 535\ \text{K}$$

Substituting the numerical values of the temperatures and γ in (8.32), we get

$$\eta_{th} = 1 - \frac{1}{1.4}\left(\frac{535 - 300}{1500 - 994}\right) = 0.668 = 66.8\%$$

Quantity of heat added per kg of air can be calculated using (8.30) as

$$\begin{aligned}\frac{Q_{in}}{m} &= C_p\,(T_C - T_B) = \frac{\gamma R}{\gamma - 1}\,(T_C - T_B)\\ &= \frac{1.4 \times 8.314}{29 \times 0.4}\,(1500 - 994)\ \text{kJ/kg} = 507.7\ \text{kJ/kg}\end{aligned}$$

The net work output per kg of air can be calculated as

$$\frac{W_{net}}{m} = \eta_{th} \times \frac{Q_{in}}{m} = 0.668 \times 507.7\ \text{kJ/kg} = 339.2\ \text{kJ/kg}$$

Comment: The cycle studied in this problem is known as the ideal Diesel cycle. It is an idealized cycle used to understand the working of an engine in which air contained in a piston-cylinder arrangement is compressed to a high temperature and ignited by injecting the fuel into the hot air, as in a 4-stroke car engine fueled by diesel. Also, notice that the efficiency of the ideal Diesel cycle operating between 1500 K and 300 K is only 66.8%, where as the Carnot efficiency for the same temperature extremes is 80% (see **Example 8.4**).

Example 8.7

In a cyclic process, air initially at 1 bar and 300 K is compressed adiabatically to reduce its volume to one tenth of the initial value. The compressed air is then heated at constant volume until its pressure increased by 50% of the value at the end of adiabatic compression. The air in then expanded adiabatically and finally cooled at constant pressure to its initial state. Sketch the cyclic process on a P-V diagram. Assuming that all processes are quasistatic and that air behaves as as ideal gas with γ =1.4, determine the thermal efficiency of the cycle.

Solution to Example 8.7

The cyclic process sketched on a P-V diagram is shown in Figure 8.10, where A → B is the quasistatic adiabatic compression, B → C is the constant-volume heating, C → D is the quasistatic adiabatic expansion, and D → A is the constant-pressure cooling.

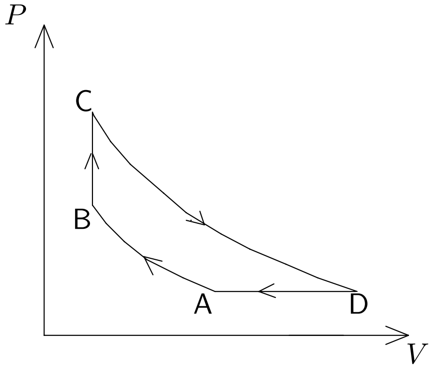

Figure 8.10 The path of the cyclic process of Example 8.7.

We can calculate η_{th} using (8.19), where Q_{in} is the amount of heat added during the constant-volume heating B → C, and Q_{out} is the amount of heat rejected during the constant-pressure cooling D → A. No heat is added or removed during the adiabatic processes A → B and C → D.

Constant-volume heat addition can be calculated using (8.13) as

$$Q_{in} = m\, C_v\, (T_C - T_B) \tag{8.36}$$

Constant-pressure heat removal can be calculated using (8.12) as

$$Q_{out} = -m\,C_p\,(T_A - T_D) = m\,C_p\,(T_D - T_A) \tag{8.37}$$

Combining (8.19), (8.36) and (8.37), we have

$$\eta_{th} = 1 - \gamma\,\left(\frac{T_D - T_A}{T_C - T_B}\right) \tag{8.38}$$

where $T_A = 300$ K, and T_B, T_C and T_D are unknown.

To determine T_B, T_C and T_D, let us tabulate all the known data at the states A, B, C and D as in Table 8.4.

A	$P_A = 1$ bar	$T_A = 300$ K	$V_A = ?$
B	$P_B = ?$	$T_B = ?$	$V_B = V_A/10$
C	$P_C = 1.5 \times P_B = ?$	$T_C = ?$	$V_C = V_B = V_A/10$
D	$P_D = P_A = 1$ bar	$T_D = ?$	$V_D = ?$

Table 8.4 Data at states A, B, C and D of the cyclic process.

Since A → B is a quasistatic adiabatic compression of an ideal gas, (7.30) can be used to find T_B as

$$T_B = \left(\frac{V_A}{V_B}\right)^{(\gamma-1)} T_A = 10^{0.4} \times 300 \text{ K} = 753.6 \text{ K}$$

Since B → C is a constant-volume process of an ideal gas, ideal gas equation can be used to find T_C as

$$T_C = \left(\frac{P_C}{P_B}\right) T_B = 1.5 \times 753.6 \text{ K} = 1130.4 \text{ K}$$

Since C → D is a quasistatic adiabatic expansion of an ideal gas, (7.30) can be used to find T_D as

$$\begin{aligned} T_D &= \left(\frac{P_D}{P_C}\right)^{(\gamma-1)/\gamma} T_C = \left(\frac{P_A}{1.5 \times P_B}\right)^{(\gamma-1)/\gamma} T_C \\ &= \left(\frac{1 \text{ bar}}{1.5 \times P_B}\right)^{(1.4-1)/1.4} \times 1130.4 \text{ K} \end{aligned}$$

where the unknown P_B could be found as

$$P_B = \left(\frac{V_A}{V_B}\right)^{\gamma} P_A = 10^{1.4} \times 1 \text{ bar} = 25.1 \text{ bar}$$

since A → B is a quasistatic adiabatic process of an ideal gas.

Therefore, we have

$$T_D = \left(\frac{1}{1.5 \times 25.1}\right)^{(1.4-1)/1.4} \times 1130.4 \text{ K} = 400.9 \text{ K}$$

Substituting the numerical values of the temperatures in (8.38), we get

$$\eta_{th} = 1 - 1.4\left(\frac{400.9 - 300}{1130.4 - 753.6}\right) = 0.625 = 62.5\%$$

8.12 Summary

- Heat does not reside in an object. Heat is simply the energy that transfers from one object to the other, driven by a temperature difference.
- It is not always necessary to provide heat to increase the temperature of a system. Work done on a system could also result in temperature increase of the system (see Example 8.1).
- Heat provided to a simple compressible closed system is related to the enthalpy increase of the system by the following:

$$Q_{in} = \Delta H - \int_{P_o}^{P_f} V\,dP - \int_{V_o}^{V_f} P\,dV - W_{in} \tag{8.3}$$

- For a simple compressible closed system undergoing a constant-pressure quasistatic process:

$$Q_{in} = \Delta H \;=\; m\int_{T_o}^{T_f} C_p\,dT$$

- For a simple compressible closed system undergoing a constant-volume process:

$$Q_{in} = \Delta U \;=\; m\int_{T_o}^{T_f} C_v\,dT$$