Chapter 7

BOUNDARY WORK

> Frank is struck - as if for the first time - by how much civilization depends on not seeing certain things and pretending others never occurred.
>
> – Francine Prose (*Amateur Voodoo*)

In this chapter we will learn to evaluate W_{in}, which is the work term in the first law of thermodynamics. Work can be of different types, such as boundary work, stirring work, electrical work, magnetic work, work of changing the surface area, and work to overcome friction. In this chapter, we will concentrate on the evaluation of boundary work.

7.1 Boundary Work in Real Life

Boundary work is done when the boundary of the system moves, causing either compression or expansion of the system. A real-life application of boundary work, for example, is found in the diesel engine which consists of pistons and cylinders as the prime component of the engine.

In a diesel engine, air is fed to the cylinder and is compressed by the upward movement of the piston. The boundary work done by the piston in compressing the air is responsible for the increase in air temperature. Onto the hot air, diesel fuel is sprayed, which leads to spontaneous self ignition of the fuel-air mixture. The chemical energy released during this combustion process would heat up the gases produced during combustion. As the gases expand due to heating, they would push the piston downwards with great force. A major portion of the boundary work done by the gases on the piston gets converted into the energy required to rotate the shaft of the engine.

The rotating shaft of the diesel engine is responsible for turning the wheels of an automobile. The rotating shaft could also be used to spin a coil between the north and south poles of an electromagnet thus generating electricity as in the generators.

7.2 Evaluation of Boundary Work

Consider, for example, compression of the gas contained in the piston-cylinder device shown in Figure 7.1.

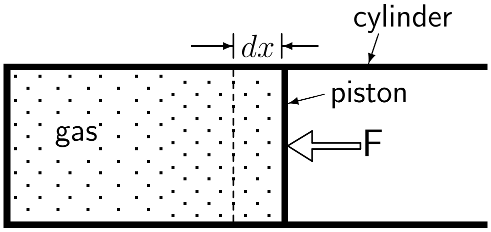

Figure 7.1 A compression process during which boundary work is done on the system by its surroundings.

Take the gas contained in the piston-cylinder device as the system. The inner surfaces of the piston and cylinder define the boundary of the system. Everything outside the boundary is the surroundings. During compression, the boundary is moved by the surroundings so as to reduce the volume of the system.

If at a certain state during the process, say, the boundary of the system is displaced by an infinitesimal (that is, a very tiny) distance dx by the application of force F as shown in Figure 7.1. The dashed line shown in the figure indicates the new position of the boundary (note that the displacement is exaggerated.) Boundary work is evaluated by determining the product between the displacement of a system boundary and the force causing the displacement in the direction of the displacement. The infinitesimal amount of boundary work done by the surroundings to compress the gas is therefore

$$dW = F\,dx \tag{7.1}$$

The external force F acting on one side of the piston, as shown in Figure 7.1, tries to compress the gas. On the other side of the piston, the gas pressure applies a force to resist the compression. If we assume that the compression process is **almost fully resisted** then the forces acting on either side of the piston become almost equal to each other. It follows then,

$$F = P\,A \tag{7.2}$$

where P is the gas pressure that is assumed to be **uniformly distributed** everywhere within the gas at any state during the compression, and A is the cross-sectional area of the piston on which the gas pressure acts.

The change in the volume of the gas, designated by dV, is related to dx by the following:

$$-\,dV = A\,dx \tag{7.3}$$

since the gas has experienced a reduction in its volume, we know that dV is a negative quantity. It is therefore a negative sign is placed in front of dV in (7.3) to be able to equate it to the product of two positive quantities, A and dx.

Eliminating F and dx from (7.1) using (7.2) and (7.3), respectively, we get

$$\begin{aligned} dW &= (P\,A)\left(\frac{-\,dV}{A}\right) \\ &= -\,P\,dV \end{aligned} \tag{7.4}$$

which is the infinitesimal amount of boundary work done **on** the system **by** its surroundings. In other words, this amount of boundary work enters the system. Incorporating this information as a subscript on dW, (7.4) is rewritten as

$$dW_{in} = -\,P\,dV \tag{7.5}$$

If the process considered were an expansion process, then the system would have moved the boundary so as to increase the volume of the system. The infinitesimal amount of boundary work done **by** the system **on** its surroundings (that is, the work leaving the system) would therefore be given by

$$dW_{out} = P\,dV \tag{7.6}$$

Integrating (7.5) or (7.6) between the initial state with volume V_o, and the final state with volume V_f, we get

$$W_{out} = -\,W_{in} = \int_{V_o}^{V_f} P\;dV \tag{7.7}$$

The total boundary work is therefore expressed in terms of system properties P and V by (7.7). It is important to observe that (7.7) can be applied to evaluate the boundary work for any substance, solid, liquid, gas or any mixture of phases, provided that the expansion or compression process occurs under almost fully resisted condition.

`Student:` Teacher, one of the assumptions you made to derive at equation (7.7) is that the system pressure P is **uniformly distributed** everywhere within the system at any state during the compression process. I don't feel comfortable with that assumption. It is because I think, during compression, the gas close to the face of the piston gets compressed more than the gas away from the face of the piston leading to nonuniform pressure distribution?

`Teacher:` Yes, if the compression process were sudden, the gas close to the face of the piston would get compressed more than the gas away from the face of the piston. The resulting pressure distribution within the system would then be far from uniform. On the other hand, if the compression or the expansion process is carried in such a manner that it is **almost fully resisted** while taking place, then it will be possible to maintain an **almost** uniform pressure distribution within the system as it undergoes the compression or expansion process.

`Student:` Teacher, could you please explain me how exactly a process is carried out under almost fully resisted condition?

`Teacher:` Yes, I will. Take a look at Figure 7.2. Let us imagine that the little unhappy faces shown in the figure represent the gas contained in the cylinder, trapped by the piston represented by the black solid bar shown in the figure. The force exerted by the gas pressure on the piston pushes the piston upwards thus causing expansion of the gas. The total weight of the piston, the person standing on the piston and the basket of tiny stones carried by him causes the piston to move downwards compressing the gas. When these two forces balance each other, the system is in equilibrium. Let us now remove a single tiny stone from the basket. Since the mass of the tiny stone removed is extremely small, the force exerted by the gas molecules on the piston to move it upwards becomes slightly larger than the force exerted to prevent the expansion of gas. The piston thus moves upwards by a very small distance allowing the gas to expand slightly. Such an expansion process is an **almost fully resisted expansion**, since it is carried out with a very small difference (an infinitesimal difference) between the force causing the expansion and the force resisting the expansion. **An almost fully resisted compression process** would be the reverse of the above process.

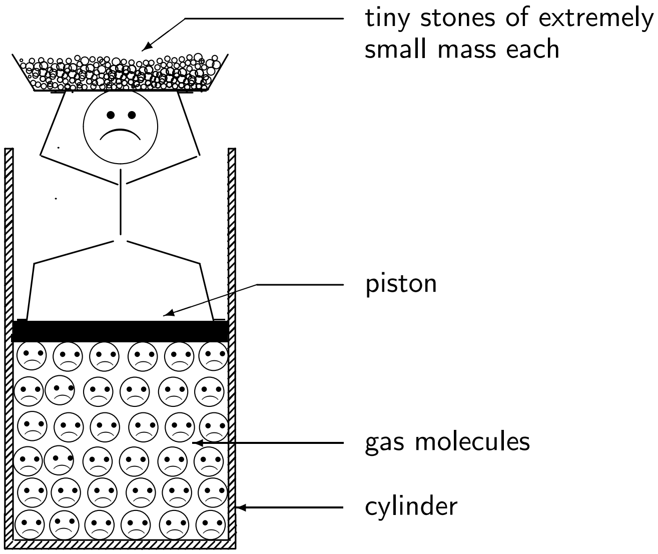

Figure 7.2 Almost fully-resisted expansion of a gas in a piston-cylinder device.

`Student:` I see that the forces acting on either side of the piston almost equal each other in an almost fully resisted process. I understand very well now how you got equation (7.2). If we had removed the stones in the basket that the person is carrying all at once, then the piston would have risen rapidly causing the gas to experience sudden expansion. Or, if we had loaded the basket with a lot more stones all at once then the piston would have moved rapidly downwards causing sudden compression of the gas. Am I right, Teacher?

`Teacher:` Yes, you are. Note that the external force F is related to a gas pressure P by (7.2) which is a property of the system, only because the compression is assumed to be almost fully resisted. If the process were not an almost fully resisted expansion or compression then the relationship given by (7.2), in general, would not hold.

`Student:` Yes, I see that. However, I still do not see how it is possible to maintain a uniform pressure distribution during such a process.

`Teacher:` Since there is only an infinitesimal difference between the force causing the expansion (or compression) and the force resisting the expansion (or compression) in an almost fully resisted process, the pressure undergoes only an infinitesimal change at a time. Any temperature change created by the expansion (or compression) would also be infinitesimal in its magnitude. An almost fully resisted process such as the one discussed above could progress only very slowly so that it would allow enough time for the infinitesimal changes experienced by the properties of the system, such as pressure, temperature, and specific volume, to be distributed almost uniformly across the system.

`Student:` Thank you, Teacher. I understand now how the uniform distribution of properties, I mean, the **almost** uniform distribution of properties, is maintained in a system when the expansion or compression process is carried out under almost fully resisted condition.

7.3 Quasistatic Process

As beginners, we will mostly be working with the special kind of process known as the **quasistatic process**, which is defined as a process whose path is composed of **near** equilibrium states. The initial and final states of any quasistatic process are equilibrium states, and all the states that a system passes through along the path of a quasistatic process are almost equilibrium states. A quasistatic process is also known as a quasi-equilibrium process.

We have seen in Section 2.9 that an equilibrium state can be represented by a point on a property diagram. Therefore, it is acceptable to represent the path of a quasistatic process, which is made up of near equilibrium states, by a curve on a property diagram, as shown in Figure 7.3.

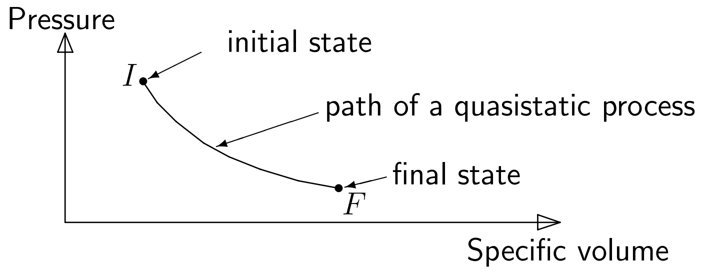

Figure 7.3 A quasistatic process with its initial state I, its final state F, and its path shown on a P-v diagram.

Consequently, it is possible to describe the path of a quasistatic process by an equation, such as $P = f(v)$ on a P-v diagram, $T = f(v)$ on a T-v diagram, and $P = f(T)$ on a P-T diagram. Moreover, since each state is almost an equilibrium state along a quasistatic process, the properties at a state along the path of a quasistatic process can be interrelated by, for example, the ideal gas equation of state.

The path followed by a non-quasistatic process consists of states that may or may not be equilibrium states, and therefore the path of a non-quasistatic process cannot be represented by a curve on a property diagram.

The fully resisted expansion or compression process, which we were introduced to in the section above, is an example of a quasistatic process since each state along a fully resisted expansion or compression is almost an equilibrium state.

7.4 Boundary Work in Real Processes

Boundary work can be evaluated using (7.7) only for processes that approximate a quasistatic process. Strictly speaking, boundary work associated with many real systems, such as the piston-cylinder arrangements of diesel engines described in Section 7.1, cannot be evaluated using the work integral of (7.7). It is because the uniform nature of the properties within the system at each and every state along the path of the process is not maintained in such systems owing to the very high speeds at which the piston moves.

Moreover, in the piston-cylinder device, for example, friction occurs between the surface of contact of the piston and cylinder. The resulting friction force on the piston acts in a direction opposite to the motion of the piston, and therefore work is done to overcome the effect of friction. The problem here, however, is the evaluation of the exact amount of work done against the friction force, which is not an easy task.

Besides, friction forces acting on the piston has the capacity to hold the piston in place until the difference between the forces acting on either side of the piston reaches a finite value. At this point, the friction could be overcome and the piston would undergo a sudden movement, which in turn would cause sudden expansion or compression of the gas contained in the piston-cylinder device. The movement of the piston caused by a finite force and the resulting sudden expansion or compression of the gas destroy the quasistatic nature of the process. Therefore, we simplify our calculation of boundary work in many real systems by making the assumption that the piston is free to move within the cylinder without friction.

If the process is not quasistatic then there is no direct way of calculating the work done by the system, or on the system. Since it is next to impossible to do calculations with non-quasistatic processes, we assume, wherever possible, that the given process is quasistatic. This assumption makes it possible for us to carry out calculations with real thermodynamic processes.

Owing to the assumptions that friction forces are absent and that the properties are almost uniformly distributed within the system, a quasistatic process sets a limit for the best performance that is expected of a real process. A device producing work produces the maximum amount of work and a device requiring work requires the minimum amount of work when they undergo quasistatic processes.

7.5 Units for Boundary Work

Since work is a form of energy, the unit for work is the same as the unit for energy. If P takes the unit Pa and V takes the unit m^3 in the work integral of (7.7), then the unit of W_{in} becomes

$$\text{Pa}\times\text{m}^3 = (\text{N/m}^2)\times\text{m}^3 = \text{N}\cdot\text{m} = \text{J}.$$

In this textbook, we will use the unit kPa for P. And, therefore, the unit of W_{in} will be kJ.

When the total volume V of (7.7) is replaced by the specific volume v, we get

$$w_{in} = -\int_{v_o}^{v_f} P\,dv \tag{7.8}$$

in which w_{in} takes the unit kJ/kg provided v is in m^3/kg. If v were the molar volume, taken in m^3/kmol, w_{in} would be in kJ/kmol. In both these cases P is in kPa.

7.6 Path Dependence of Boundary Work

Consider the gas contained in the piston-cylinder device of Figure 7.1 once again. Imagine that we could push the piston slowly to decrease the volume occupied by the gas while maintaining the quasistatic nature of the process. Let us say that we measured the volume occupied by the gas and the corresponding pressure of the gas at certain times during the compression process, and the data so gathered were plotted on a P-V diagram as shown by the dots of Figure 7.4. Connect the data points by a smooth curve as in the figure. The curve so obtained represents the path of the process between its initial state (o) and its final state (f) on the P-V diagram. The equation describing the curve will provide the relationship between P and V along the path of the process.

Integrating along the path of the process on the P-V diagram of Figure 7.4 in the increasing direction of V, we get

$$\int_{V_f}^{V_o} PdV$$

The value of the above integral is simply the area of the shaded region shown in Figure 7.4. Interchanging the limits in the above integral, we get

$$-\int_{V_o}^{V_f} PdV$$

which, according to (7.7), is W_{in}. That is, the area under the curve representing the path of a compression process on a P-V diagram gives W_{in}, which is the boundary work done on the system by its surroundings.

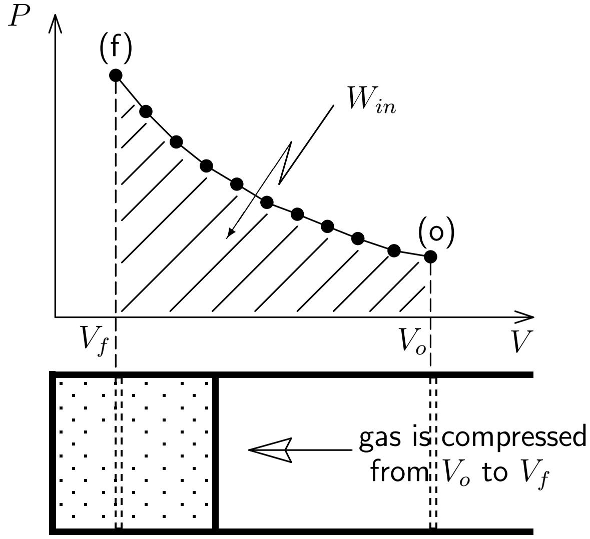

Figure 7.4 When the process takes place in the direction of decreasing volume (that is, compression), the area of the shaded region gives W_{in}.

If the process considered is an expansion process, then the volume of the system increases from V_o to V_f as shown in Figure 7.5. Integrating along the path of the process on the P-V diagram in Figure 7.5, we get

$$\int_{V_o}^{V_f} PdV$$

which, according to (7.7), gives W_{out}. That is, the area under the curve representing the path of an expansion process on a P-V diagram gives W_{out}, which is the boundary work done by the system on its surroundings.

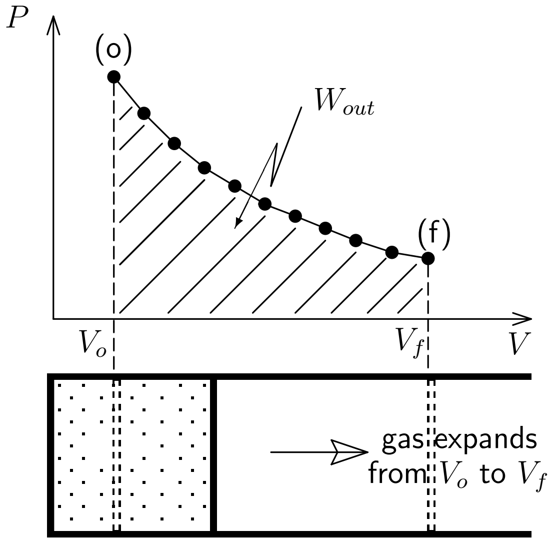

Figure 7.5 When the process takes place in the direction of increasing volume (that is, expansion), the area of the shaded region gives W_{out}.

It is important to note that boundary work given by the area of the shaded region in Figure 7.4 or in Figure 7.5 will change if the path of the process, described by the curve on the P-V diagram, changes. This means that even if the initial and final states of a process are fixed, the boundary work would be different for the different paths followed by a process. Therefore, we say that boundary work is **path dependent**.

7.7 Other Forms of Work

Boundary work is just one mode of work transfer between the system and its surroundings. There are also other modes of work, such as shaft work, surface work, electrical work, magnetic work, electrical polarization work, etc. Shaft work is a form of work associated with a rotating shaft which is a prime component in stirrers, pumps, compressors, turbines, diesel and petrol engines. Shaft work per unit time is given by $2\pi\dot{n}\tau$, where $\dot{n}$ is the number of revolutions made by the shaft per unit time and τ is the constant value of the torque applied to the shaft. When τ takes the unit N · m and $\dot{n}$ takes the unit revolutions per second, shaft work takes the unit J/s, which is W.

In this book we deal only with simple compressible systems (defined in Section 2.7). Since such systems do not experience electrical, magnetic and surface tension effects, we will not get into details of how the work needed to change surface area, electrical work, magnetic work or electrical polarization work are evaluated. For details of the evaluation of these modes of work, please refer to the following textbooks on thermodynamics: (i) Holman, J.P. 1988 Thermodynamics, 4th Edition, McGraw-Hill International Editions. (ii) Wark, K. 1989 Thermodynamics, 5th Edition, McGraw-Hill International Editions.

7.8 Worked Examples

Example 7.1

The following statements are incorrect. Find the error in each statement.

(a) For a gas contained in a closed system changing from one state to another state, $\int_o^f PdV$ is the same along any path as long as the process is quasistatic.

(b) Boundary work is always given by $\int_o^f PdV$.

(c) All forms of work transfer are zero for a constant volume process.

(d) A closed, adiabatic system of gas expanding from an initial pressure P_o to a final pressure P_f against a constant force applied by its surroundings is a quasistatic process.

Solution to Example 7.1

(a) The boundary work done in a quasistatic process is evaluated using $\int_o^f PdV$. But the value of $\int_o^f PdV$, and thus the amount of boundary work, change depending on the path of the chosen process. Therefore, even if the process is quasistatic, the value of $\int_o^f PdV$ depends on the path of the quasistatic process considered.

(b) Boundary work of a quasistatic process is given by $\int_o^f PdV$. Boundary work associated with a non-quasistatic process is nearly impossible to calculate, since such a process could have a nonuniform distribution of the properties within the system. Therefore, boundary work is not always given by $\int_o^f PdV$.

(c) The boundary work associated with a constant volume quasistatic process is zero, since dV is identically zero in the work integral $\int_o^f PdV$ used to determine the boundary work of a quasistatic process. Even if the given constant volume process is not a quasistatic process, there is neither boundary work nor frictional work involved with the process, since the boundary does not move in a constant volume process. However, a constant volume process may receive stirring work, and therefore it is incorrect to assume that all forms of work transfer are zero for a constant volume process.

(d) For the given adiabatic expansion process to qualify as a quasistatic process, it should have been carried out under conditions of infinitesimal difference in the force causing the expansion and the force resisting the expansion. In the given process, the force resisting the expansion remains constant, whereas the force causing the expansion changes from P_oA to P_fA, where A is the cross-sectional area on which the pressure acts. Therefore, the expansion is carried out under finite difference between the force causing the expansion and the force resisting the expansion. Therefore, the given process is not quasistatic.

Example 7.2

Determine the boundary work (a) for a quasistatic constant-pressure process, and (b) for a quasistatic constant-volume process. Pressures and volumes at the initial and the final states of the process are given by (P_o, V_o) and (P_f, V_f), respectively.

Solution to Example 7.2

(a) A quasistatic constant-pressure process: We employ (7.7) to evaluate the boundary work since the given process is quasistatic. Since it is also a constant pressure process, $P_o = P_f = P$, say. Therefore, we get from (7.7)

$$W_{out} = -\,W_{in} = \int_{V_o}^{V_f} P\,dV = P\,\int_{V_o}^{V_f} dV = P\,(V_f - V_o) \tag{7.9}$$

for a quasistatic constant-pressure process.

In case we deal with ideal gases, then the ideal gas equation of state gives $P_oV_o = nRT_o$ and $P_fV_f = nRT_f$, where n is the amount of gas in the closed system, R is the universal gas constant, and T_o and T_f are the temperatures at the initial and the final states, respectively. Therefore, (7.9) yields

$$W_{out} = -\,W_{in} = n\,R\,(T_f - T_o) \tag{7.10}$$

for a quasistatic constant-pressure process of an ideal gas.

(b) A quasistatic constant-volume process: Since the given process is quasistatic, (7.7) is employed to evaluate the boundary work. Since it is also a constant volume process, $dV = 0$ always. Therefore, (7.7) gives

$$W_{out} = -\,W_{in} = \int_{V_o}^{V_f} P\,dV = 0 \tag{7.11}$$

That is to say the boundary work for a quasistatic constant-volume process is zero, since dV of a constant volume process is identically zero.

Example 7.3

The piston-cylinder device of Figure 7.1 contains hydrogen, say, which expands from an initial state given by (500 kPa, 0.10 m^3) to a final state given by (230 kPa, 0.18 m^3) in two steps. In the first step, the gas is expanded to the volume of 0.18 m^3 while maintaining the pressure constant at 500 kPa. In the second step, the gas pressure is decreased to 230 kPa while maintaining the volume constant at 0.18 m^3. Determine the work done by hydrogen gas.

Solution to Example 7.3

Analytical method: Let us assume that the given process takes place quasistatically. Boundary work done by the gas along the constant-pressure quasistatic process is given by (7.9) as

$$W_{out}\big|_P = P\,(V_f - V_o)$$

Substituting, $P = 500$ kPa, $V_o = 0.10$ m^3 and $V_f = 0.18$ m^3, we get

$$W_{out}\big|_P = 500\,(0.18 - 0.10)\ \text{kPa}\times\text{m}^3 = 40.0\ \text{kJ}$$

Boundary work done by the gas along the constant-volume quasistatic process is given by (7.11) as

$$W_{out}\big|_V = 0$$

The total boundary work done by the gas in the two-step process is, therefore,

$$W_{out} = W_{out}\big|_P + W_{out}\big|_V \quad = 40.0\ \text{kJ} + 0 \quad = 40.0\ \text{kJ}$$

Graphical method: Figure 7.6 shows the path of the process, represented by the curve IEF on a P-V diagram. The initial state and the final state are marked by points I and F, respectively. The intermediate state where the process switches from a constant-pressure process to a constant-volume process is marked by point E. The area under the curve IEF is

$$500\ \text{KPa}\times(0.18\text{-}0.10)\ \text{m}^3 = 40\ \text{kJ},$$

which is the work done by hydrogen (see Figure 7.5). Observe that the work evaluated by the graphical method is the same as the work evaluated by the analytical method, which is how it should be.

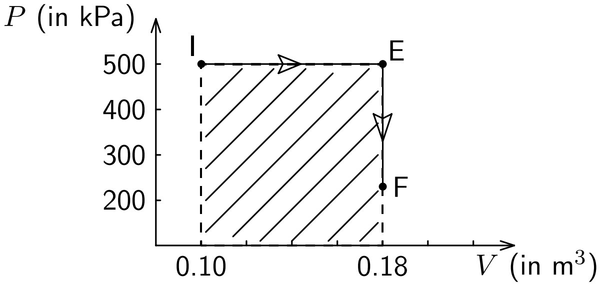

Figure 7.6 Path of the process of Example 7.3, shown on a P-V diagram.

Example 7.4

Hydrogen in the piston-cylinder device of **Example 7.3**, let us say, expands by pushing the piston in a way that the pressure of the gas decreases continuously while the volume of the gas increases continuously. The data collected during the expansion process are tabulated below. Determine the boundary work done by hydrogen.

Pressure (in kPa)	500	393	320.5	268.6	230
Volume (in m^3)	0.10	0.12	0.14	0.16	0.18

Solution to Example 7.4

The boundary work done by hydrogen could be evaluated using the work integral of (7.7) if we know how P changes with V along the path of the process. Let us plot the pressure-volume data collected during the expansion process on a P-V diagram. The data are represented by dots marked I, E_1, E_2, E_3 and F in Figure 7.7.

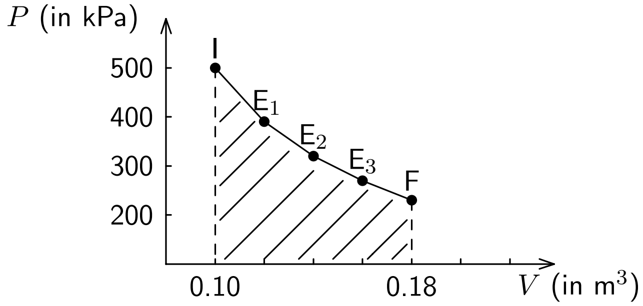

Figure 7.7 Path of the process of Example 7.4, shown on a P-V diagram.

The area under the curve $IE_1E_2E_3F$ represents the boundary work done by hydrogen (see Figure 7.5). It is therefore possible to determine this area by plotting the curve $IE_1E_2E_3F$ on a graph sheet and then by counting the squares under the curve on the graph sheet.

Alternatively, the equation of the curve that best fits the data points in Figure 7.7 would give us the equation describing the path of the process. Let us first try the equation

$$P V^k = C \tag{7.12}$$

where k and C are constants to be determined from the experimental data using the procedure outlined below. Taking the natural logarithm of (7.12) yields

$$\ln P = -k\ \ln V + \ln C$$

which shows that a plot of $\ln P$ versus $\ln V$ gives a straight line having a slope -k and an intercept $\ln C$.

Now let us go back to the experimental data on P and V, find the natural logarithms of P and V, and plot the data on a $\ln P$ versus -$\ln V$ diagram, as shown in Figure 7.8. We see in Figure 7.8 that the data points are well fitted by a straight line. Therefore, we conclude that (7.12) accurately describes the path of the process.

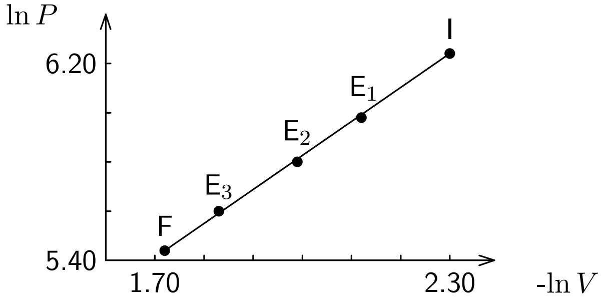

Figure 7.8 $\ln P$ versus -$\ln V$ plot of the data in Example 7.4.

Substituting P from (7.12) in (7.7), we get

$$W_{out} = \int_{V_o}^{V_f} \left(\frac{C}{V^k}\right)\ dV$$

which can be integrated, for cases in which $k \neq 1$, to yield

$$W_{out} = C\left[\frac{V^{-k+1}}{-k+1}\right]_{V_o}^{V_f} = C\ \left(\frac{V_f^{-k+1} - V_o^{-k+1}}{-k+1}\right) \qquad (7.13)$$

From the slope and the intercept of the straight line in Figure 7.8, we can find that k = 1.32 and C = 23.9 when P is in kPa and V in m^3. Substituting these values and the data V_o = 0.10 m^3 and V_f = 0.18 m^3 in (7.13), we get

$$W_{out} = 23.9\ \left(\frac{0.18^{-0.32} - 0.10^{-0.32}}{-0.32}\right) = 26.8 \text{ kJ}$$

`Teacher:` Note that both the processes of **Example 7.3** and **Example 7.4** have the same initial and final states. The path of the processes, of course, are different from each other as shown in Figure 7.6 and Figure 7.7. The work done by hydrogen also differs exhibiting the path-dependence of the work integral given by (7.7).

`Student:` Yes, I noted that. Teacher, I have a question. Suppose that the data points on the $\ln P$ versus -$\ln V$ plot of the **Solution to Example 7.4** could not have been fitted by a straight line, How could we solve the problem then?

`Teacher:` We should have then tried another form of equation in place of (7.12). However, it is sometimes impossible to describe the path of the process by an equation. In such cases, the work transfer is determined from evaluating the area under the path of the process on a P-V diagram plotted on a graph sheet.

Example 7.5

The initial volume V_o of a cyclic process is the same as the final volume V_f. Therefore, your friend argues, the work integral (7.7) used to evaluate the boundary work of a quasistatic process reduces to zero owing to the upper and the lower limits of the integral of (7.7) becoming one and the same. Consequently, your friend concludes that the boundary work associated with a quasistatic cyclic process of a closed system is always zero. Explain to your friend, using an appropriate example, why he is wrong.

Solution to Example 7.5

For a constant-volume process, boundary work is zero since dV of a constant volume process is identically zero throughout the entire process. The process of this example is not a constant volume process having $dV = 0$ always, but a cyclic process which has identically the same volumes at the initial and the final states of the process. An example of a quasistatic cyclic process is shown on the P-V diagram of Figure 7.9 by the path $A \to B \to C \to D \to A$.

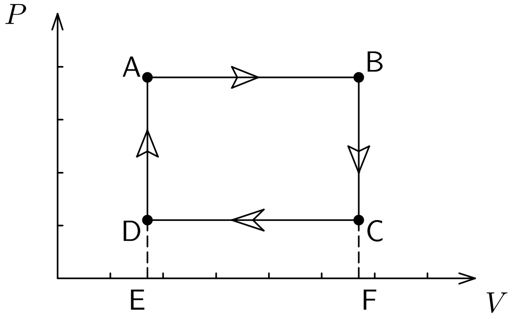

Figure 7.9 An example of a quasistatic cyclic process.

Since the constant-pressure quasistatic process $A \to B$ occurs in the direction of increasing volume, the area under the path $A \to B$ in Figure 7.9, enclosed by the rectangle $ABFE$, gives $(W_{out})_{A \to B}$ (see Figure 7.5). Therefore,

$$(W_{out})_{A \to B} = P_A\,(V_B - V_A)$$

The constant-pressure quasistatic process $C \to D$ occurs in the direction of decreasing volume, and therefore the area under the path $C \to D$ in Figure 7.9, enclosed by the rectangle $CDEF$, gives $(W_{in})_{C \to D}$ (see Figure 7.4). Therefore,

$$(W_{in})_{C \to D} = P_C\,(V_C - V_D)$$

Since $B \to C$ and $D \to A$ are constant-volume quasistatic processes,

$$(W_{out})_{B \to C} = 0 \quad \text{and} \quad (W_{out})_{D \to A} = 0$$

as in (7.11). Note that the constant-volume quasistatic paths $B \to C$ and $D \to A$ have zero areas under them on the P-V diagram of Figure 7.9.

Putting all the information above together, we calculate the net work transfer as follows:

$$\begin{aligned}(W_{out})_{net} &= (W_{out})_{A \to B} + (W_{out})_{B \to C} + (W_{out})_{C \to D} + (W_{out})_{D \to A} \\ &= P_A\,(V_B - V_A) + 0 - P_C\,(V_C - V_D) + 0\end{aligned}$$

From Figure 7.9, we see that $V_A = V_D$ and $V_B = V_C$, and therefore we reduce the above to the following:

$$(W_{out})_{net} = (P_A - P_C)\,(V_B - V_A)$$

which is in fact the area enclosed by the rectangle $ABCD$ in Figure 7.9. It is obvious therefore that the work transfer in a process with identical initial and final volumes, as in a cyclic process, need not always be zero.

`Teacher:` The above example shows that the work exchange between the system and its surroundings in a quasistatic cyclic process is given by the area enclosed by the path of the process on the P-V diagram. Please note that the cyclic process shown in Figure 7.9 is in the clockwise direction, and that the area enclosed by the path of the process represents the work done by the system on its surroundings, i.e. W_{out},

`Student:` Yes, Teacher. I have noted that.

`Teacher:` If the cyclic process considered were in the anti-clockwise direction on the P-V diagram then the area enclosed by the path of the process would have represented the work done on the system by its surroundings, i.e. W_{in}.

`Student:` Okay, Teacher. I will work that out myself.

Example 7.6

Determine the boundary work associated with a quasistatic isothermal process of an ideal gas. Pressures and volumes at the initial and the final states of the process concerned are given by (P_o, V_o) and (P_f, V_f), respectively.

Solution to Example 7.6

In an isothermal process, the temperature remains constant. An ideal gas satisfies the ideal gas equation, $P\,V = n\,R\,T$. Therefore, an isothermal process of an ideal gas satisfies the following condition:

$$P\,V = n\,R\,T = C \tag{7.14}$$

where C is a constant since T is a constant. In this process P changes hyperbolically with V. Since the process is taken as a quasistatic process, we evaluate the work done on the system using (7.7) as follows:

$$W_{in} = -\int_{V_o}^{V_f} P\, dV \;=\; -\int_{V_o}^{V_f} \left(\frac{C}{V}\right) dV \;=\; -C\,\ln\frac{V_f}{V_o}$$

Applying $PV =$ constant for the initial and final states, that is $P_oV_o = P_fV_f$, we get

$$W_{in} = -C\,\ln\left(\frac{V_f}{V_o}\right) \;=\; -C\,\ln\left(\frac{P_o}{P_f}\right) \tag{7.15}$$

for a **quasistatic isothermal process of an ideal gas**. The value of C in (7.15) is determined using (7.14) as follows:

$$C = P_oV_o = P_fV_f = nRT \tag{7.16}$$

Example 7.7

One kmol of an ideal gas originally at 300 K and 1 bar is heated at constant pressure to a temperature of 400 K and then compressed isothermally to a volume equal to its initial volume. Show this two-step process on a P-V diagram, and calculate the net work exchange of the system with its surroundings. State your assumptions clearly.

Solution to Example 7.7

During the first step of the process, the ideal gas at 300 K and 1 bar is heated at constant pressure to a temperature of 400 K. Since the temperature increases at constant pressure, the volume of the gas increases as well. If we take this constant pressure process to be quasistatic, then it can be shown on the P-V diagram of Figure 7.10 by the path $A \rightarrow B$. Point A represents the initial state having $P_A = 1$ bar and $T_A = 300$ K, and point B represents the intermediate state having $P_B = 1$ bar and $T_B = 400$ K.

During the second step, the gas is compressed isothermally to a volume equal to its initial volume. That is, the temperature remains constant at 400 K and the volume decreases to its initial value, V_A. Since the volume decreases at constant temperature, the pressure of the gas increases. Taking this isothermal process to be quasistatic, its path, described by the hyperbola $PV = nRT =$

constant, can be shown by the curve $B \rightarrow C$ on the P-V diagram of Figure 7.10. Point C represents the final state of the process having $T_C = 400$ K and $V_C = V_A$.

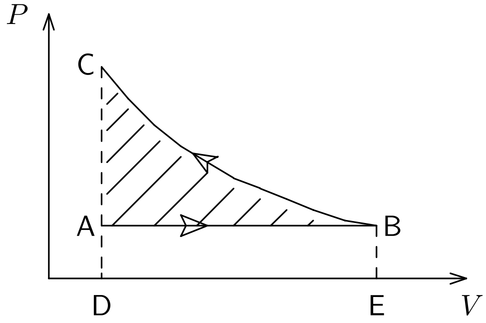

Figure 7.10 The two-step process of Example 7.7 shown on a P-V diagram, where the shaded area shows the net work done on the system.

Since path $A \rightarrow B$ is in the direction of increasing volume, the area $ABED$ under path $A \rightarrow B$ gives W_{out} (see Figure 7.5). Therefore, we get

$$(W_{out})_{A\rightarrow B} = P_A\,(V_B - V_A)$$

where we know that $P_A = 1$ bar, but we do not know the volumes at A and B. Since the temperatures at A and B are known, the volumes at A and B can be eliminated from the above equation using the ideal gas equation of state as follows:

$$(W_{out})_{A\rightarrow B} = P_B\,V_B - P_A\,V_A = n\,R\,(T_B - T_A)$$

in which we have used the fact that $P_A = P_B$. Since $n = 1$ kmol, $T_A = 300$ K and $T_B = 400$ K, we get

$$\begin{aligned}(W_{out})_{A\rightarrow B} &= (1\ \text{kmol})\ (8.314\ \text{kJ/kmol}\cdot\text{K})\ (400\ \text{K} - 300\ \text{K}) \\ &= 831.4\ \text{kJ} \qquad (7.17)\end{aligned}$$

Since path $B \rightarrow C$ is in the direction of decreasing volume, the area $BCDE$ under path $B \rightarrow C$ gives W_{in} (see Figure 7.4). Since $B \rightarrow C$ is a quasistatic isothermal process, we could calculate the work exchange using (7.15) as follows:

$$(W_{in})_{B\rightarrow C} = -\,C\,\ln\left(\frac{V_C}{V_B}\right) = -\,C\,\ln\left(\frac{P_B}{P_C}\right) \qquad (7.18)$$

To determine the volume ratio (V_C/V_B) or the pressure ratio (P_B/P_C) of (7.18), let us use the fact that the mass of ideal gas contained satisfies the ideal gas equation of state at the states represented by points A, B and C. We therefore get

$$\frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} = \frac{P_C V_C}{T_C}$$

which becomes

$$\frac{(1 \text{ bar})\, V_A}{300 \text{ K}} = \frac{(1 \text{ bar})\, V_B}{400 \text{ K}} = \frac{P_C V_A}{400 \text{ K}}$$

The above yields $P_C = 4/3$ bar. Substituting the values of P_C and P_B in (7.18), we get

$$(W_{in})_{B\to C} = -\,C \ln(3/4) \tag{7.19}$$

where C is calculated using (7.16) as follows:

$$C = nRT_B = (1 \text{ kmol})\ (8.314 \text{ kJ/kmol}\cdot\text{K})\ (400 \text{ K}) = 3326 \text{ kJ}$$

Thus, (7.19) gives

$$(W_{in})_{B\to C} = -\,3326 \ln(3/4) \text{ kJ} = 956.8 \text{ kJ} \tag{7.20}$$

Combining (7.17) and (7.20), we get the net work done on the system by the surroundings as follows:

$$(W_{in})_{net} = -(W_{out})_{A\to B} + (W_{in})_{B\to C} = (-831.4 + 956.8) \text{ kJ} = 125.4 \text{ kJ}$$

Example 7.8

Obtain an expression for the boundary work associated with steam undergoing a quasistatic isothermal process.

Solution to Example 7.8

In the case of an ideal gas undergoing a quasistatic isothermal process, the P-V path of the process can be described by the simple equation PV = constant. And, therefore the boundary work associated can be given by the simple and straight forward expression (7.15), as proved in the **Solution to Example 7.6**.

In the case of steam undergoing a quasistatic isothermal process, however, the P-V path of the process cannot be described by a simple equation. And, therefore the boundary work associated cannot be given by a simple and straight forward expression. However, if the path of the process could be sketched on a P-V diagram then it would be possible to graphically determine the area under the path between the initial and the final states of the process, and hence the work required.

Example 7.9

Determine the boundary work associated with an adiabatic process executed by a closed system.

Solution to Example 7.9

There is no heat transfer in an adiabatic process, and thus $Q_{in} = 0$. The first law of thermodynamics applied to closed system therefore gives

$$(W_{in})_{adiabatic} = \Delta U \tag{7.21}$$

which is one instant in which the work transfer W_{in}, a path dependent function, is equated to ΔU, change in a system property.

Whether the given adiabatic process is quasistatic or not, (7.21) can be used to determine the boundary work associated with a closed system containing ideal gas or steam or any other substance. The work given by (7.21) is known as **adiabatic work**. It is important to note that the adiabatic work can be evaluated without knowing the P-V path of the process concerned.

Example 7.10

An ideal gas of 0.1 kmol expands adiabatically such that its temperature reduces from 400°C to 150°C. Calculate the boundary work done by the gas assuming γ to be 1.4.

Solution to Example 7.10

Since the given process is adiabatic, the work transfer can be calculated using the expression for adiabatic work given by (7.21). Since we are dealing with ideal gas, (7.21) becomes

$$(W_{in})_{adiabatic} = n \int_{T_o}^{T_f} C_v \, dT \tag{7.22}$$

Since C_v is a constant and since $C_v = R/(\gamma - 1)$, (7.22) reduces to the following:

$$\begin{aligned} (W_{in})_{adiabatic} &= n\, C_v\, (T_f - T_o) \quad = n\ (R/\gamma - 1)\ (T_f - T_o) \qquad (7.23) \\ &= 0.1 \times (8.314/0.4) \times (150 - 400) \text{ kJ} \\ &= -\, 519.6 \text{ kJ} \end{aligned}$$

The boundary work done by the ideal gas is 519.6 kJ.

Example 7.11

Superheated steam of 0.1 kmol expands adiabatically from 5 bar and 400°C to 1 bar and 150°C. Calculate the boundary work done by the steam.

Solution to Example 7.11

Since the given process is adiabatic, the work transfer can be calculated using the expression for adiabatic work given by (7.21). Since we are dealing with steam, (7.21) should be expanded to

$$(W_{in})_{adiabatic} = m\,(u_f - u_o)$$

where $m = 0.1 \times 18$ kg, and u_f and u_i are the specific internal energies of steam at the final state at 1 bar 150°C and at the initial state at 5 bar and 400°C, respectively.

From a Superheated Steam Table, we can find that $u_f = 2583$ kJ/kg and that $u_i = 2963$ kJ/kg. Therefore,

$$(W_{in})_{adiabatic} = 1.8 \times (2583 - 2963) \text{ kJ} = -684.0 \text{ kJ}$$

The boundary work done by steam is 684.0 kJ.

Example 7.12

Develop a pressure-volume relationship describing the path of a quasistatic adiabatic process of an ideal gas.

Solution to Example 7.12

Consider a mass of ideal gas undergoing a quasistatic adiabatic process. Let us suppose that this mass of gas receives dW_{in} amount of boundary work. Since the process is quasistatic, the boundary work shall be given by (7.5) as

$$dW_{in} = -P\,dV \tag{7.24}$$

The work entering this adiabatic system increases the internal energy of the system. And, the differential increase in the specific or molar internal energy of an ideal gas is given by (5.7) as $du = C_v\,dT$. The differential increase in the internal energy of the system is therefore

$$dU = m\,C_v\,dT \tag{7.25}$$

Since no heat is transferred between the system and its surroundings in an adiabatic process, the first law applied to an adiabatic closed system relates dW_{in} to dU by

$$dW_{in} = dU \tag{7.26}$$

Combining (7.24), (7.25) and (7.26), we get

$$-\,P\,dV = m\,C_v\,dT \tag{7.27}$$

for an ideal gas undergoing a quasistatic adiabatic process.

Using the ideal gas equation of state, T in (7.27) can be replaced by $(P\,V/m\,R)$, as follows:

$$\begin{aligned} -P\,dV &= m\,C_v\,d\left(\frac{P\,V}{m\,R}\right) \\ &= \frac{C_v}{R}\left(P\,dV + V\,dP\right) \end{aligned}$$

The above can be rearranged to give

$$-\,(R + C_v)\,P\,dV = C_v\,V\,dP \tag{7.28}$$

Since $C_p = C_v + R$ for an ideal gas, (7.28) becomes

$$-C_p\,P\,dV = C_v\,V\,dP$$

which, using $\gamma = C_p/C_v$, can be rearranged to give

$$\frac{dP}{P} = -\gamma \, \frac{dV}{V}$$

Integrating the above assuming that γ remains a constant, we get

$$\ln P = -\gamma \, \ln V + \text{ constant}$$

which can be rearranged to give

$$P\, V^{\gamma} = C \tag{7.29}$$

where C is a constant. Equation (7.29) describes the path of a quasistatic adiabatic process of an ideal gas on a P-V diagram.

To find the equation describing the path of the quasistatic adiabatic process of an ideal gas on a T-V diagram, substitute $P = m\,RT/V$ in (7.29). We get

$$\frac{m\,RT}{V}\, V^{\gamma} = C$$

which yields the required equation as follows:

$$T\, V^{(\gamma-1)} = \frac{C}{mR} = \text{ constant} \tag{7.30}$$

To find the equation describing the path of the quasistatic adiabatic process of an ideal gas on a P-T diagram, substitute $V = mRT/P$ in (7.29). That gives

$$P\left(\frac{m\,RT}{P}\right)^{\gamma} = C$$

which yields the required equation as follows:

$$\frac{T}{P^{(\gamma-1)/\gamma}} = \frac{C^{1/\gamma}}{(nR)} = \text{ constant} \tag{7.31}$$

Example 7.13

Twenty moles of an ideal gas undergoes a quasistatic process in which the gas expands adiabatically from $P_o = 10$ bar and $T_o = 425$ K to $P_f = 1$ bar and $T_f = 220$ K. Determine the isentropic exponent γ, and evaluate the work done by the system.

Solution to Example 7.13

Since the given process is a quasistatic adiabatic process of an ideal gas, the process shall be described by (7.29). Since the initial and final states of the process are identified by P and T, not by P and V, we shall describe the process by (7.31). Since the initial and the final states of the process satisfy (7.31), we get

$$\frac{T_o}{P_o^{(\gamma-1)/\gamma}} = \frac{T_f}{P_f^{(\gamma-1)/\gamma}}$$

Since $P_o = 10$ bar, $T_o = 425$ K, $P_f = 1$ bar and $T_f = 220$ K, we get

$$\frac{425}{10^{(\gamma-1)/\gamma}} = \frac{220}{1^{(\gamma-1)/\gamma}}$$

from which the numerical value of γ could be evaluated as follows:

$$\begin{aligned} 10^{(\gamma-1)/\gamma} &= 425/220 \\ \frac{\gamma-1}{\gamma} &= \frac{\ln(425/220)}{\ln(10)} \\ \frac{1}{\gamma} &= 1 - 0.286 \\ \gamma &= 1.4 \end{aligned}$$

Since the process is an adiabatic process of an ideal gas with constant C_v, (7.23) is used to determine the work transfer as follows:

$$W_{in} = \frac{n\,R}{\gamma-1}(T_f - T_o) = \frac{0.02 \times 8.314}{1.4-1} \times (220-425) \text{ kJ} = -85.2 \text{ kJ}$$

Thus, the work done by the ideal gas is 85.2 kJ.

Example 7.14

A quasistatic **polytropic process** process of a simple compressible gaseous system is described by $PV^k =$ constant, where k is known as the **polytropic constant**. Obtain an expression for the work done by the system undergoing a quasistatic polytropic process in terms of the pressures and volumes at the initial and final states of the process.

Solution to Example 7.14

Starting from (7.7), the work done on the system during a quasistatic polytropic process may be given by

$$\begin{aligned} W_{in} &= -\int_{V_o}^{V_f} \left(\frac{C}{V^k}\right) dV \\ &= -C\left(\frac{V_f^{-k+1} - V_o^{-k+1}}{-k+1}\right) \quad \text{for } k \neq 1 \end{aligned} \tag{7.32}$$

The constant C in (7.32) is removed by the following procedure. Since $P\,V^k = C$ is applicable at both the initial and the final states of the process, we get

$$C = P_o V_o^k = P_f V_f^k \tag{7.33}$$

Using (7.33) in (7.32), we get

$$\begin{aligned} W_{in} &= \frac{C\,V_f^{-k+1} - C\,V_o^{-k+1}}{k-1} \quad \text{for } k \neq 1 \\ &= \frac{P_f V_f^k\,V_f^{-k+1} - P_o V_o^k\,V_o^{-k+1}}{k-1} \quad \text{for } k \neq 1 \\ &= \frac{P_f V_f - P_o V_o}{k-1} \quad \text{for } k \neq 1 \end{aligned} \tag{7.34}$$

which can be used to evaluate the boundary work in a **quasistatic polytropic process**.

If the substance contained in the system were an ideal gas then the ideal gas equation $PV = nRT$ could be used to simplify (7.34) to

$$W_{in} = \frac{n\,R\,(T_f - T_o)}{k-1} \quad \text{for } k \neq 1 \tag{7.35}$$

which can be used to evaluate the boundary work in a **quasistatic polytropic process of an ideal gas**.

Example 7.15

A frictionless piston-cylinder device contains 0.1 kg of superheated steam at 6 bar and 450°C. Steam is now expanded quasistatically along a quasistatic polytropic path until the pressure and temperature fall to 1 bar and 200°C, respectively. Determine the work done by the steam.

Solution to Example 7.15

Since the steam undergoes a quasistatic polytropic process, we can use (7.34) to determine the work transfer during the process. Since 0.1 kg of steam is expanded from 6 bar and 450°C to 1 bar and 200°C, (7.34) becomes

$$W_{in} = (0.1\ \text{kg}) \left[\frac{(100\ \text{kPa})\, v_f - (600\ \text{kPa})\, v_o}{k-1} \right] \tag{7.36}$$

where v_f and v_o are the specific volumes at the final and initial states, respectively, and k is the polytropic constant.

From a Superheated Steam Table, we can find that $v_o = 0.5528\ \text{m}^3/\text{kg}$ and $v_f = 2.173\ \text{m}^3/\text{kg}$, and therefore (7.36) becomes

$$W_{in} = -\,\frac{11.44}{k-1}\ \text{kJ} \tag{7.37}$$

where k is an unknown.

There is only one way to find k which is to use the fact that the properties at the initial and the final states of the process satisfy the equation describing the path of the process, which is $P\,V^k = \text{constant}$. That leads to the following:

$$\begin{aligned}
P_o\,V_o^k &= P_f\,V_f^k \\
P_o\,v_o^k &= P_f\,v_f^k \\
6 \times (0.5528)^k &= 1 \times (2.173)^k \\
\left(\frac{2.173}{0.5528}\right)^k &= 6 \\
k &= \frac{\ln 6}{\ln(2.173/0.5528)} \quad = 1.31
\end{aligned}$$

Using $k = 1.31$ in (7.37), we get $W_{in} = -\ 38.1$ kJ. The work done by steam during the quasistatic polytropic expansion is 38.1 kJ.

Example 7.16

Consider a quasistatic expansion process described by $P = aV + b$, where a and b are constants. The pressure is 100 kPa and the volume is 0.2 m^3 at the initial state of the process, and the pressure is 900 kPa and the volume is 1 m^3 at the final state. Calculate the work done by the gas during the process.

Solution to Example 7.16

Since the given process is quasistatic, the boundary work can be evaluated using (7.7). Since the process is described by $P = aV + b$, (7.7) gives

$$W_{out} = \int_{V_o}^{V_f} (aV + b)\, dV \;=\; a\left(\frac{V_f^2 - V_o^2}{2}\right) + b\,(V_f - V_o)$$

The problem statement gives $V_o = 0.2$ m^3 and $V_f = 1$ m^3, and therefore the above expression reduces to

$$\begin{aligned} W_{out} &= a\left[\frac{(1 - 0.04)\ \text{m}^6}{2}\right] + b\,[1 - 0.2]\ \ \text{m}^3 \\ &= a\,(0.48\ \text{m}^6) + b\,(0.8\ \text{m}^3) \end{aligned} \tag{7.38}$$

The values of the constants a and b are not given, and there is only one way to evaluate the constants a and b, which is to use the fact that the initial state given by (100 kPa, 0.2 m^3) and the final state given by (900 kPa, 1 m^3) should satisfy the process described by $P = aV + b$. Therefore, we get the following two equations:

$$100\ \text{kPa} = a\,(0.2\ \text{m}^3) + b \qquad \text{and} \qquad 900\ \text{kPa} = a\,(1\ \text{m}^3) + b$$

solving which we find $a = 1000$ kPa/m^3 and $b = -100$ kPa. Substituting the values of a and b in (7.38), we get

$$W_{out} = 1000\ \text{kPa/m}^3\left(0.48\ \text{m}^6\right) + (-100\ \text{kPa})\,(0.8)\ \ \text{m}^3 \;=\; 400\ \text{kJ}$$

The work done by the gas during the process is 400 kJ.

`Student:` Teacher, the expansion process of **Example 7.16** is described by $P = aV + b$, which means pressure increases as the volume increases. I would expect the pressure to decrease with increasing volume......

`Teacher:` It is not surprising that you expect the pressure to decrease with increasing volume. Because that's what happens in most cases. However, there are situations in which pressure increases with increasing volume. One such case is discussed in **Example 7.17** where the expansion of a gas is restrained by the force exerted by a spring.

Example 7.17

A frictionless piston-cylinder device initially contains air at 150 kPa and 0.2 m^3. At this state, a linear spring (that is, a spring for which the force F is proportional to displacement x) is touching the piston but exerts no force on it. The air is now heated slowly to a final state of 0.5 m^3 and 600 kPa. Plot the process on a P-V diagram. Hence, or otherwise, determine the work done by the air stating clearly all the assumptions made.

Of the total work done by air, find the work done against the variable force exerted by the spring, and the work done against the constant force due to the weight of the piston and the atmospheric pressure. Neglect the weight of the spring in comparison to the weight of the piston.

Solution to Example 7.17

The initial, intermediate, and the final states of the process are shown in Figure 7.11. At the initial state, the spring just touches the piston and exerts no force on the piston. As the air in the piston-cylinder device is heated slowly, it expands slowly pushing the piston against the spring to compress it. This way, the expansion of air is almost fully resisted by the spring provided that the frictional effects are negligible, and can therefore be approximated to a quasistatic process. The work done by air can thus be evaluated using the work integral given by (7.7), provided we know how the pressure of air (P) changes with volume of air (V) in the given process.

Since the expansion of air is almost fully resisted by the spring, the forces acting on the piston are almost in balance according to

$$P\,A \approx P_o\,A \;+\; M\,g + \;k\,x \qquad (7.39)$$

where A is the cross-sectional area of the piston on which the pressure P of air acts, P_o is the atmospheric pressure, $M\,g$ is the weight of the piston, $k\,x$ is the force exerted by the spring when the spring is compressed by a length x, and k is the spring constant. The above force balance is written at the intermediate state shown in Figure 7.11. Since the compressed length of the spring is x and the volume of air is V at the intermediate state, as shown in Figure 7.11, we get

$$\text{volume increase} \;=\; V \;-\; V_o \;=\; A\,x \tag{7.40}$$

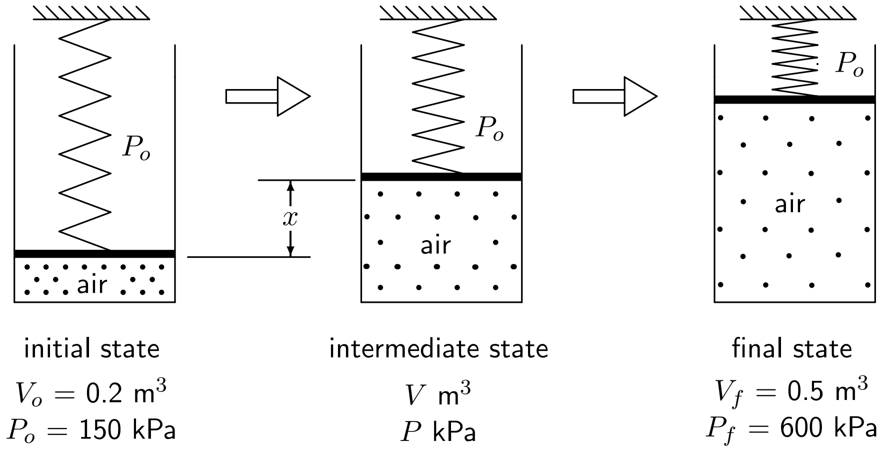

Figure 7.11 The initial, intermediate and the final states of the process given in Example 7.17.

Combining (7.39) and (7.40), we get

$$P = P_o \;+\; \frac{M\,g}{A} + \frac{k}{A}\left(\frac{V - V_o}{A}\right) \tag{7.41}$$

Equation (7.41) can be readily rearranged to give

$$P = mV \;+\; c \tag{7.42}$$

which describes a straight line on a P-V diagram since the slope m and intercept c, given by

$$m = \frac{k}{A^2} \qquad \text{and} \qquad c = P_o \;+\; \frac{M\,g}{A} - \frac{k}{A^2}V_o$$

are constants. If m and c are known, the straight line of (7.42) describing the process can be sketched on a P-V diagram. We do not know k, A, P_o and M, and therefore are unable to calculate m and c.

We do however know the coordinates of the initial state (o) and the final state (f) of the process, and they are shown on the P-V diagram of Figure 7.12. A straight line drawn through the initial and the final states, as shown in Figure 7.12, gives the path of the process given by (7.42) on the P-V diagram.

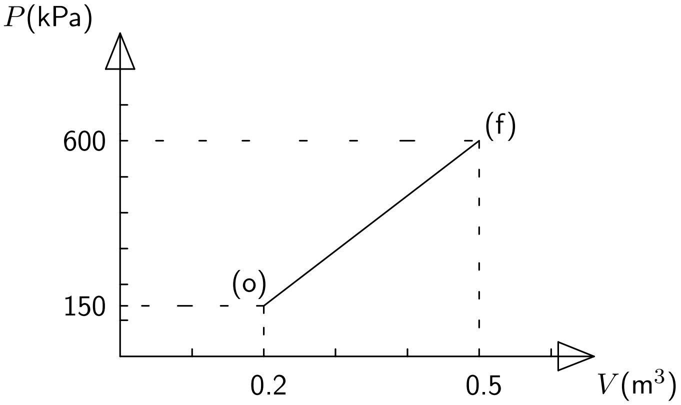

Figure 7.12 Process given in Example 7.17 shown on a P-V diagram.

The boundary work done by air is the area under the straight line and is calculated as follows:

$$W_{out} = \left(\frac{150+600}{2}\right)(0.5-0.2) \ \ \text{kPa}\cdot\text{m}^3 \ = \ 112.5 \ \text{kJ} \qquad (7.43)$$

Here, we used a graphical method to calculate the work done by air. Alternatively, we can directly evaluate the work integral of (7.7) using P of (7.42) once we know the slope m and intercept c, which of course can readily be obtained from the P-V diagram of Figure 7.12.

Of the total work done by air, we must now find out how much is against the spring and how much is against the constant force exerted by the weight of the piston and the atmospheric pressure, which is $(P_o\,A + M\,g)$. The work done by air to push against this constant force can be evaluated as follows:

Work done by air against the constant force

$$\begin{aligned} &= \ \text{(constant force)} \cdot \text{(distance moved)} \\ &= \ (P_o\,A + M\,g)\left(\frac{V_f - V_o}{A}\right) \ = \ \left(P_o + \frac{M\,g}{A}\right)(V_f - V_o) \\ &= \ \left(P_o + \frac{M\,g}{A}\right)(0.5 - 0.2) \ \ \text{m}^3 \end{aligned}$$

We now need to know the value of $(P_o\,A + M\,g)$. Let us get back to (7.41) and substitute the properties at the initial state, which are $P = P_o = 150$ kPa and $V = V_o = 0.2$ m^3. Then, we get

$$P_o \;+\; \frac{M\,g}{A} = 150 \text{ kPa}$$

Thus, the work done by air against the force exerted by the weight of the piston and the atmospheric pressure $= (150 \text{ kPa} \times 0.3 \text{ m}^3) = 45$ kJ.

The work done by air against the variable force exerted by the spring is therefore the difference between the total work done by air and the work done by air against the constant force. Thus, the work done by air to push the spring is (112.5 - 45) kJ = 67.5 kJ.

Example 7.18

Rework **Example 7.17** with a nonlinear spring for which $F \propto x^2$. Measurements taken at an intermediate point of the expansion process showed that the pressure of air is 500 kPa when its volume is 0.4 m^3. Determine the total work done by air.

Solution to Example 7.18

For a nonlinear spring, we get

$$P\,A \approx P_o\,A \;+\; M\,g + \;k\,x^2\;, \tag{7.44}$$

in the notation used in the **Solution to Example 7.17**. Using the fact that the volume increase is $(V - V_o) = A\,x$ in (7.44), we get

$$P = P_o \;+\; \frac{M\,g}{A} + \frac{k}{A}\left(\frac{V - V_o}{A}\right)^2 \tag{7.45}$$

Equation (7.45) can be rearranged to give

$$P = a\,V^2 \;+\; b\,V \;+\; c \tag{7.46}$$

which takes a parabolic shape on a P-V diagram with a, b and c as constants.

Even though we do not know k, A, P_o and M to find the constants a, b and c, we know that the curve represented by (7.46) on the P-V diagram passes through the initial state (0.2 m^3, 150 kPa), the intermediate state (0.4 m^3, 500 kPa), and the final state (0.5 m^3, 600 kPa). Forcing (7.46) to satisfy these three points, we get $a = -2500$ kPa/m^6, $b = 3250$ kPa/m^3, and $c = -400$ kPa.

The work done by air is evaluated by substituting (7.46) in the work integral of (7.7) as follows:

$$\begin{aligned} W_{out} &= \int_{V_o}^{V_f} (a\,V^2 \;+\; b\,V \;+\; c)\,dV \\ &= a\left(\frac{V_f^3 - V_o^3}{3}\right) + b\left(\frac{V_f^2 - V_o^2}{2}\right) + c\,(V_f - V_o) \end{aligned}$$

Substituting the numerical values of a, b and c along with $V_f = 0.5$ m^3 and $V_o = 0.2$ m^3, we get $W_{out} = 124$ kJ.

Example 7.19

A piston-cylinder device initially contains steam at 200 kPa, 150°C, and 0.10 m^3. At this state, a linear spring for which $F \propto x$ is touching the piston but exerts no force on it. Heat is now slowly added to the steam, causing the pressure and the volume to rise to 300 kPa and 0.12 m^3, respectively. Determine the work done by the steam and the heat supplied to the steam.

Solution to Example 7.19

Whether the system contains ideal gas or steam, it could be proven using the method discussed in the **Solution to Example 7.17** that the P-V relationship of the given process is given by

$$P = mV \;+\; c \tag{7.47}$$

Since the initial and the final states of the steam satisfies (7.47), we get the following equations:

$$200 \text{ kPa} = m(0.10 \text{ m}^3) + c \quad \text{and} \quad 300 \text{ kPa} = m(0.12 \text{ m}^3) + c$$

which gives $m = 5000$ kPa/m^3 and $c = -300$ kPa. The P-V relationship of (7.47) therefore becomes

$$P = 5000V \;-\; 300 \tag{7.48}$$

where P is in kPa and V is in m^3.

The work done by steam is evaluated by substituting (7.48) in the work integral of (7.7) as follows:

$$\begin{aligned} W_{out} &= \int_{0.10\ \text{m}^3}^{0.12\ \text{m}^3} (5000V - 300)\, dV \\ &= 5000 \left(\frac{0.12^2 - 0.10^2}{2} \right) - 300\,(0.12 - 0.10) \quad = 5\ \text{kJ} \end{aligned}$$

Heat supplied to the steam, denoted by Q_{in}, is found by applying the first law of thermodynamics to the steam as follows:

$$Q_{in} = \Delta U - W_{in} = \Delta U + W_{out} = \Delta U + 5\ \text{kJ} \tag{7.49}$$

of which ΔU is not yet known.

Since we deal with steam, we could expand ΔU as

$$\Delta U = m\,(u_f - u_o) \tag{7.50}$$

where m is the mass of steam and u_o and u_f are the specific internal energies at the initial and final states.

The initial state of steam is at 200 kPa and 150°C, and it could be found from a Superheated Steam Table that $v_o = 0.9602$ m^3/kg and $u_o = 2578$ kJ/kg. Since the initial volume of steam is 0.10 m^3, we can calculate the mass of steam as follows:

$$m = \frac{0.10\ \text{m}^3}{0.9602\ \text{m}^3/\text{kg}} = 0.1041\ \text{kg}$$

The final state of steam is at 300 kPa. Since the final volume is 0.12 m^3, we can calculate the specific volume of steam at the final state as follows:

$$v_f = \frac{0.12\ \text{m}^3}{0.1041\ /\text{kg}} = 1.153\ \text{m}^3/\text{kg}$$

From a Superheated Steam Table, it could be found that at 300 kPa and 1.153 m^3/kg, the specific internal energy $u_f = 3093$ kJ/kg.

Substituting the numerical values of m, u_o and u_f in (7.50), we get

$$\Delta U = (0.1041\ \text{kg})\,(3093 - 2578)\ \text{kJ/kg} = 53.6\ \text{kJ}$$

Substituting the value of ΔU in (7.49), we get $Q_{in} = 53.6\ \text{kJ} + 5\ \text{kJ} = 58.6\ \text{kJ}$. Thus, the heat supplied to the steam is 58.6 kJ and the work done by the steam is 5 kJ.

Example 7.20

The container shown in Figure 7.13 contains 10 kg of oxygen at a temperature of 350°C. A constant force of 20 kN is applied on the face of the piston having a cross-sectional area of 0.05 m^2. The atmospheric pressure is 101.3 kPa. Heat is added slowly to oxygen until its temperature rises to 400°C. Determine the work done by oxygen, and the amount of heat delivered to oxygen. Assume that the piston is free to move. The average value of C_v for oxygen can be taken as 0.66 kJ/kg · K, and the molar mass be taken as 32 kg/kmol.

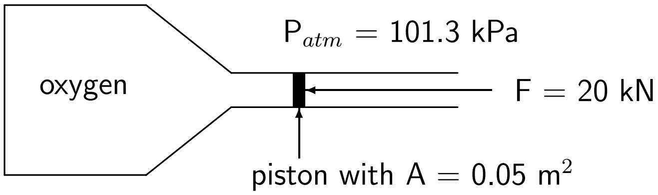

Figure 7.13 Oxygen contained in the piston-cylinder device.

Solution to Example 7.20

Let the oxygen in the container be the closed system. We need to determine the work done by the oxygen, and the amount of heat delivered to it. Owing to heat addition, the temperature of oxygen increases from 350°C to 400°C. The temperature increase causes the volume of oxygen to increase pushing the piston away, and we need to evaluate the boundary work associated with the volume change. Since heat is added slowly, let us assume that the oxygen pushes the piston slowly enough for the process to be assumed quasistatic. The boundary work can then be evaluated using the work integral of (7.7), provided we find out how the pressure P of oxygen varies with its volume V.

Since the piston moves freely, the pressure force acting on the inner face of the piston will almost be balanced by the applied force of 20 kN and the atmospheric pressure acting on the outer face of the piston. Thus, a force balance on the piston gives

$$P = \frac{20 \text{ kN}}{0.05 \text{ m}^2} + 101.3 \text{ kPa} = 501.3 \text{ kPa} = \text{ a constant}$$

This means that the process is a constant pressure process, and the work integral of (7.7) becomes

$$W_{out} = P\,(V_f - V_o)$$

in which $P = 501.3$ kPa, but V_f and V_o are unknowns. Since oxygen is assumed to behave as an ideal gas, the above expression for work can be converted to

$$\begin{aligned} W_{out} &= m\,R\,(T_f - T_i) \\ &= \left(\frac{10\text{ kg}}{32\text{ kg/kmol}}\right)(8.314\text{ kJ/kmol}\cdot\text{K})\,(400-350)\text{ K} \;= 130\text{ kJ} \end{aligned}$$

The amount of heat delivered to oxygen, given by Q_{in}, is found by applying the first law of thermodynamics to oxygen as follows:

$$Q_{in} = \Delta U - W_{in} = \Delta U + W_{out} = \Delta U + 130\text{ kJ} \tag{7.51}$$

of which ΔU is unknown. Assuming that the oxygen behaves like an ideal gas, we get

$$\Delta U = m\,C_v\,(T_f - T_o) = (10\text{ kg})\,(0.66\text{ kJ/kg}\cdot\text{K})\,(400-350)\text{ K} \;= 330\text{ kJ}$$

Substituting the value of ΔU in (7.51), we get

$$Q_{in} = 330\text{ kJ} + 130\text{ kJ} = 460\text{ kJ}$$

Thus, the heat supplied to oxygen in the container is 460 kJ and the work done by oxygen is 130 kJ.

Example 7.21

The piston-cylinder device shown in Figure 7.14, contains 0.1 kg of air (molar mass = 29 kg/kmol; C_v = 0.717 kJ/kg·K). Initially, air is at 100 kPa and 27°C, and the piston rests on stops. Air is heated until its volume increases by 20% of the initial volume. An air pressure of 200 kPa is required to raise the piston. Stating the assumptions made, mark the path of the process on a P-V diagram, and determine the total work done by air and the total heat supplied to air.

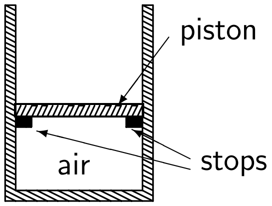

Figure 7.14

Solution to Example 7.21

The initial pressure of air is 100 kPa, and the air pressure required to raise the piston is 200 kPa. Thus, the piston will rest on the stops maintaining the volume of air constant until the pressure of air becomes 200 kPa. This part of the process is therefore a constant volume process, during which the pressure and temperature of air increase. Assuming the process to be quasistatic, the path of the process is shown on the P-V diagram of Figure 7.15 by the constant volume line $A \rightarrow B$. Point A represents the initial state of the process with P_A = 100 kPa and T_A = 27°C = 300 K. Point B represents the state at which the air pressure becomes 200 kPa at constant volume, and P_B = 200 kPa and V_B = V_A.

The air pressure required to raise the piston is 200 kPa. This means that the weight of the piston and the atmospheric pressure acting on the piston together exert a net force of 200 kPa which keeps the piston resting on the stops until the air pressure reaches 200 kPa. A small increase in the air pressure at this point would cause the piston to rise. The consequent volume increase would help to maintain the pressure of the air to remain at 200 kPa, provided the piston rises quasistatically. The air pressure would then remain at 200 kPa, and the volume and temperature of air would increase as the air continues to receive heat at constant pressure, until the volume becomes 1.2 times its initial value. The path of this process is shown in Figure 7.15 by the constant pressure line $B \rightarrow C$. Point C represents the final state of the process, at which $P_C = P_B$ = 200 kPa and V_C = 1.2 V_B.

The path of the combined process is given by $A \rightarrow B \rightarrow C$.

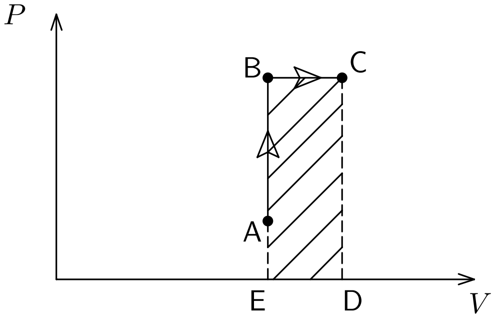

Figure 7.15 The process of Example 7.21 shown on a P-V diagram.

The total work done by air on its surroundings is represented by the shaded area $BCDE$ since the process is in the direction of increasing volume (see Figure 7.5). Therefore, we get

$$(W_{out})_{total} = P_B\,(V_C - V_B) = (200\ \text{kPa})\,(1.2\,V_B - V_B) \tag{7.52}$$

where V_B is unknown.

Of the states represented by A, B and C, we know the pressure and temperature only at A. We also know that the mass of air in the piston-cylinder device is 0.1 kg. Therefore, volume V_A is found by applying $P\,V = m\,R\,T$ to state A as follows:

$$V_A = \frac{m\,R\,T_A}{P_A} = \frac{(0.1\ \text{kg})\ (8.314/29\ \ \text{kJ/kg}\cdot\text{K})\ (300\ \text{K})}{100\ \text{kPa}} = 0.086\ \text{m}^3$$

Since $V_B = V_A$, we get $V_B = 0.086\ \text{m}^3$. Thus, (7.52) gives

$$(W_{out})_{total} = (200\ \text{kPa})\,(0.2 \times 0.086\ \text{m}^3) = 3.44\ \text{kJ}$$

We have yet to determine the total heat supplied to air, which can be done by applying the first law of thermodynamics to the entire process as follows:

$$\begin{aligned}(Q_{in})_{total} &= (\Delta U)_{total} - (W_{in})_{total} \\ &= m\,C_v\,(T_C - T_A) + (W_{out})_{total} \\ &= 0.1 \times 0.717 \times (T_C - 300)\ \text{kJ} + 3.44\ \text{kJ} \qquad (7.53)\end{aligned}$$

where T_C is unknown.

Temperature T_C is found by applying the ideal gas equation of state at states A and C as follows:

$$\begin{aligned}\frac{P_C\,V_C}{T_C} &= \frac{P_A\,V_A}{T_A} \\ \frac{(200\ \text{kPa})\,(1.2\,V_B)}{T_C} &= \frac{(100\ \text{kPa})\,V_B}{300\ \text{K}} \\ T_C &= 720\ \text{K} \qquad (7.54)\end{aligned}$$

Combining (7.53) and (7.54), we get

$$(Q_{in})_{total} = (0.1)\,(0.717)\,(720 - 300)\ \text{kJ} + 3.44\ \text{kJ} = 33.55\ \text{kJ}$$

Example 7.22

A rigid box made of thermally non-conducting material is divided into two compartments of volume 0.5 m^3 each by a thin, strong, thermally non-conducting diaphragm. Initially, one compartment contains nitrogen at 5 bar and 300 K and the other is a vacuum. When the diaphragm is raptured, the nitrogen will rush to fill the entire box. Determine the temperature and the pressure of nitrogen at the final equilibrium state, and the boundary work done by nitrogen.

Solution to Example 7.22

The initial and the final equilibrium states are shown in Figure 7.16. At the initial state, nitrogen exerts a pressure force equivalent to 5 bar on one side of the diaphragm. The other side experiences no force on it since it faces a vacuum. Once the diaphragm is raptured, nitrogen expands to fill the entire box. The expansion of nitrogen is unrestrained since there is no force acting on the moving part of the boundary of the nitrogen. At the final equilibrium state, the nitrogen occupies the entire box.

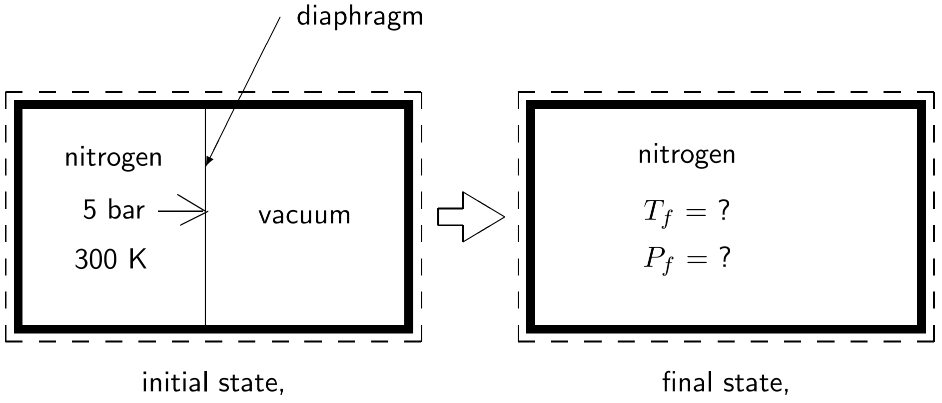

Figure 7.16 The initial and final states of the system of Example 7.22.

Choose the system as the entire rigid box and all of its content. Since the rigid box is made up of non-conducting material, no heat enters or leaves the system. Since the box is rigid, no work enters or leaves the system. Thus, by the first law, the internal energy content of the system remains a constant.

The contribution of the box and the diaphragm towards the internal energy change is negligible, and the vacuum makes no contribution, and therefore the internal energy of nitrogen remains a constant. If we assume that the nitrogen behaves as an ideal gas, then the temperature of nitrogen remains a constant. Therefore, the final equilibrium temperature of nitrogen is the same as its initial temperature. That is, $T_f = 300$ K.

The final equilibrium pressure of nitrogen can be found by the application of the ideal gas equation of state at the initial and final states of nitrogen as follows:

$$\frac{P_f\,(1\ \mathsf{m}^3)}{300\ \mathsf{K}} = \frac{(5\ \mathsf{bar})\,(0.5\ \mathsf{m}^3)}{300\ \mathsf{K}}$$

which gives $P_f = 2.5$ bar.

To evaluate the boundary work done by nitrogen, let us take the nitrogen as the system, and apply the first law of thermodynamics to it. The nitrogen is enclosed by non-conducting material and vacuum, and therefore no heat has entered or left the nitrogen during the process. The internal energy of nitrogen has been shown to remain constant. Therefore, the first law of thermodynamics applied to nitrogen gives that no work is done by nitrogen.

`Student:` Teacher, a part of the boundary of nitrogen in **Example 7.22** has moved all the way to the opposite wall of the box. How could then the boundary work done by nitrogen be zero?

`Teacher:` We have just proved that the boundary work done by nitrogen is zero. Haven't we?

`Student:` Yes, Teacher, we have. But, please, could we evaluate the boundary work done by nitrogen using (7.7), instead of using the first law as we have just done?

`Teacher:` Well, as stated above, nitrogen has undergone an unrestrained expansion. That is, the expansion did not take place under almost fully-resisted conditions. Therefore, the expansion of nitrogen is not quasistatic, and we cannot use (7.7) to evaluate the boundary work done by the nitrogen.

`Student:` Teacher, the boundary of nitrogen has moved during the expansion of nitrogen. Therefore, nitrogen should have done some boundary work. I am very sure of that. How did we get zero? Oh... Teacher, I am going crazy.

`Teacher:` Cool down, dear Student. I agree that the boundary of nitrogen has moved during the expansion of nitrogen. But, the boundary has moved with absolutely no force resisting its movement since nitrogen has expanded into a vacuum. It is, therefore, I say that nitrogen does not do any work when it expands into a vacuum. It is as simple as that.

`Student:` Yes, I see the point now. It is simple indeed, but very strange.

`Teacher:` It might appear strange, but very true. Remember, boundary work is always done against a force. By the way, you know that it is more difficult to climb a mountain than to walk a flat land. It is because we go against the gravitational force when climbing the mountain. If there were no gravitational force acting on us, we would have floated to the top of Mount Everest with ease. Wouldn't we?

`Student:` Yes, we would.

Example 7.23

The piston-cylinder device in Figure 7.17 contains 0.2 kmol of helium. The weight of the piston and the atmospheric pressure acting on it exerts a constant force equivalent to 3 bar pressure on helium. The stops shown prevent the piston from being pushed out by helium at 300 K and 8 bar. With the stops removed, helium expands adiabatically to 3 bar. The molar mass of helium is taken as 4 kg/kmol and its γ as 1.667. Determine the work transfer and the change in the internal energy of the system.

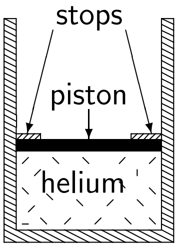

Figure 7.17

Solution to Example 7.23

First, let us see if the given process can be assumed to be quasistatic. Recall that we have learned that the force causing the expansion and the force resisting the expansion should differ only by a very small amount at any given time in a process that is quasistatic. In the given process, helium gas expands from 8 bar pressure to 3 bar pressure against a force that is constant at 3 bar. The large difference between the force causing the expansion and the force resisting the expansion makes the process non-quasistatic. Since the process is not quasistatic, we cannot use (7.7) to evaluate the boundary work.

However, since the given process is adiabatic,

$$W_{in} = \Delta U = n\, C_v\, (T_f - T_o) \tag{7.55}$$

can be used to evaluate the work transfer for helium, which behaves as an ideal gas. The amount of helium n is 0.2 kmol, C_v can be evaluated according to $C_v = R/(\gamma - 1)$ = 8.314/(1.667 −1) = 12.5 kJ/kmol · K. The initial temperature T_o is 300 K, and the final temperature T_f is to be found.

Ideal gas equation of state applied to the final state gives

$$P_f\, V_f = n\, R\, T_f,$$

where the final pressure P_f, the amount of helium n, and the gas constant R are known. Had we known the final volume V_f, we could have found T_f. Since we do not know the final volume, the ideal gas equation of state is of no help to find T_f.

We know the initial pressure P_o, the initial temperature T_o and the final pressure P_f. Therefore, it is tempting to use the relationship, derived from (7.31),

$$\frac{T_f}{P_f^{(\gamma-1)/\gamma}} = \frac{T_o}{P_o^{(\gamma-1)/\gamma}}$$

to determine T_f. We should NOT do that because the given process is not quasistatic even though it is an adiabatic process of an ideal gas.

At this point it appears that the work transfer involved in this problem cannot be found, which is however not true. Let us now try an approach that is very different from those discussed above. Let us neglect the friction and other dissipative effects involved in the process. Then, we can say that the work done by the expanding helium is done only to push the piston and the atmospheric pressure acting on the piston. Since the piston and the atmospheric pressure acting on the piston exert a constant force equivalent to 3 bar pressure on helium, we can say that the helium works against a constant force, given by the 3 bar pressure multiplied by the cross-sectional area of the piston.

The work done by the helium can therefore be calculated using

$$\begin{aligned} W_{out} &= \text{(3 bar)} \times \text{(cross-sectional area of the piston)} \\ &\qquad\qquad \times \text{(the distance moved by the piston)} \\ &= \text{(3 bar)} \times \text{(the increase in the volume of helium)} \\ &= \text{(3 bar)} \times (V_f - V_o) \end{aligned}$$

Since we do not know the initial and the final volumes of the above expression, replace these volumes by pressures and temperatures using the ideal gas equation of state as

$$W_{out} = \text{(3 bar)} \times n\,R\left(\frac{T_f}{P_f} - \frac{T_o}{P_o}\right) \tag{7.56}$$

of which we once again do not know the value of T_f.

Even though work transfer cannot be evaluated using either (7.55) or (7.56), because we do not know T_f, these two equations can be combined to find T_f as follows:

$$n\,C_v\,(T_f - T_o) = -\text{ (3 bar) } \times n\,R\left(\frac{T_f}{P_f} - \frac{T_o}{P_o}\right)$$

Substituting all known numerical values, we get

$$n\,\frac{R}{1.667-1}\,(T_f - 300\text{ K}) = -\text{ (3 bar) } \times nR\left(\frac{T_f}{3\text{ bar}} - \frac{300\text{ K}}{8\text{ bar}}\right)$$

which can be solved to give $T_f = 225$ K. Using this in (7.55), we get

$$W_{in} = (0.2\text{ kmol}) \times \frac{8.314\text{ kJ/kmol}\cdot\text{K}}{1.667-1} \times (225-300)\text{ K} = -187\text{ kJ}$$

The work done by the system on its surroundings is therefore 187 kJ. Since the system is adiabatic, the change in internal energy of the system is −187 kJ. The internal energy of the system has decreased because the system has done work on its surroundings under adiabatic conditions.

Example 7.24

A rigid cylinder contains a ‘floating’ piston. Initially, it divides the cylinder in half, and on each side of the piston the cylinder holds 0.01 kmol of the same ideal gas at 200°C and 100 kPa. Using an electrical heater installed on side A of the cylinder, heat is added slowly until the pressure in side A reaches 150 kPa. The piston and the cylinder are perfect thermal insulators with negligible heat capacity. Determine

(a) the final volume of gas on side B,
(b) the work done by the piston on the gas on side B, and
(c) the amount of heat supplied by the electrical heater.

The specific heats of the gas are constant and have the values $C_v = 12.56$ kJ/kmol · K and $C_p = 20.88$ kJ/kmol · K.

Solution to Example 7.24

The 'floating' piston is assumed to be free to move within the cylinder without friction. Therefore, the forces acting on either side of the piston equal each other when the piston is in equilibrium. Thus, at the initial and final states of the system, the pressure on side A equals the pressure on side B, as shown in Figure 7.18, which shows all the data for the problem. Heat added to side A causes the gas on side A to expand pushing the piston outwards to compress the gas on side B. Since, heat is added slowly to side A, the piston moves slowly. In addition, the 'floating' piston is assumed to have friction-free movement. It is therefore safe to assume that both the expansion of gas on side A and the compression of gas on side B are quasistatic processes. Since the piston and the cylinder are perfect heat insulators, the gas on side B shall undergo an adiabatic process.

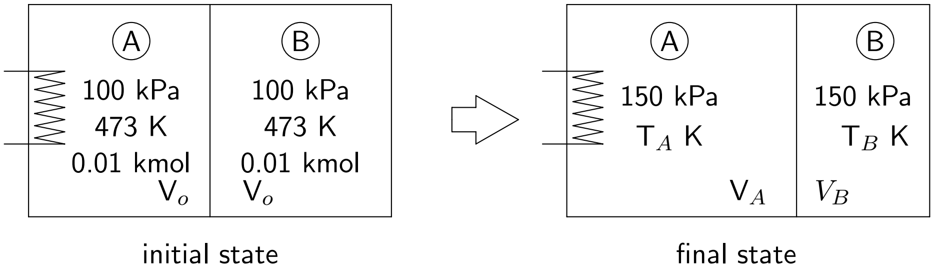

Figure 7.18 Initial and final states of Example 7.24.

(a) Determine the final volume of the gas on side B.

To determine V_B, the volume of the gas on side B at the final state, we could use the fact that the gas on side B is an ideal gas, and that it undergoes a quasistatic adiabatic process. We then know that the pressure of the gas on side B changes with its volume according to (7.29). Therefore, we get

$$(100 \text{ kPa})\, V_o^{\gamma} = (150 \text{ kPa})\, V_B^{\gamma}$$

Using $\gamma = C_p/C_v = 20.88/12.56 = 1.66$, the above expression can be simplified to

$$\frac{V_B}{V_o} = \left(\frac{100}{150}\right)^{1/1.66} = 0.78 \tag{7.57}$$

We can evaluate the initial volume V_o with the help of the ideal gas equation of state as follows:

$$V_o = \frac{(0.01 \text{ kmol})\,(8.314 \text{ kJ/kmol}\cdot\text{K})\,(300 \text{ K})}{(100 \text{ kPa})} = 0.249 \text{ m}^3 \tag{7.58}$$

Combining (7.57) and (7.58), we get

$$V_B = (0.78)\,(0.249 \text{ m}^3) = 0.194 \text{ m}^3 \tag{7.59}$$

which is the volume of the gas on side B at the final state.

(b) Determine the work done by the piston on the gas on side B.

To determine the work done by the piston on the gas on side B, that is $(W_{in})_B$, let us apply the first law of thermodynamics to the gas on side B as

$$(Q_{in})_B + (W_{in})_B = (\Delta U)_B$$

Since the gas on side B is an ideal gas that undergoes an adiabatic process, the above equation simplifies to

$$(W_{in})_B = n_B\, C_{vB}\, (T_B - 473 \text{ K}) \; = 0.01 \times 12.56 \times (T_B - 473) \text{ kJ} \tag{7.60}$$

To determine T_B, let us apply the ideal gas equation of state to the gas on side B as follows:

$$\frac{(100 \text{ kPa})(V_o)}{473 \text{ K}} = \frac{(150 \text{ kPa})(V_B)}{T_B}$$

which gives

$$T_B = \left(\frac{150}{100}\right)\left(\frac{V_B}{V_o}\right)(473 \text{ K})$$

Substituting V_B/V_o from (7.57) in the above, we get

$$T_B = \left(\frac{150}{100}\right)(0.78)\,(473 \text{ K}) = 553 \text{ K} \tag{7.61}$$

Substituting the value of T_B in (7.60), we get

$$(W_{in})_B = 0.01 \times 12.56 \times (553 - 473) \text{ kJ} = 10 \text{ kJ} \tag{7.62}$$

which is the work done by the piston on the gas on side B.

(c) Determine the amount of heat supplied by the electrical heater.

Let us say that the amount of heat supplied to the gas on side A by the electrical heater is $(Q_{in})_A$. To determine this, we may apply the first law of thermodynamics to the gas on side A as

$$(Q_{in})_A + (W_{in})_A = (\Delta U)_A$$

which can be rewritten as

$$(Q_{in})_A = (\Delta U)_A + (W_{out})_A \tag{7.63}$$

Since the piston is assumed to move without friction, the work done by the gas on side A in pushing the piston outwards is exactly the same as the work done by the piston on the gas on side B to compress it. Therefore,

$$(W_{out})_A = (W_{in})_B$$

which converts (7.63) to

$$(Q_{in})_A = (\Delta U)_A + (W_{in})_B \tag{7.64}$$

The gas on side A is 0.01 kmol of an ideal gas with $C_v = 12.56$ kJ/kmol K and $(W_{in})_B$ is known from (7.62). When this information is used in (7.64), we get

$$(Q_{in})_A = [0.01 \times 12.56 \times (T_A - 473) + 10] \text{ kJ} \tag{7.65}$$

To find T_A, let us apply the ideal gas equation of state to the gas in side A as

$$T_A = \left(\frac{150}{100}\right)\left(\frac{V_A}{V_o}\right)(473 \text{ K}) \tag{7.66}$$

We can determine V_A in the following manner. Since the total volume of gases at the final state is the same as the total volume of gases at the initial state, we get

$$V_A + V_B = 2\,V_o$$

Using V_B/V_o from (7.57), the above equation can be rearranged to give

$$V_A/V_o = 2 - V_B/V_o = 2 - 0.78 = 1.22$$

Substituting the above in (7.66) gives $T_A = 866$ K, and using this value of T_A in (7.65), we get

$$(Q_{in})_A = 0.01 \times 12.56 \times (866 - 473) + 10 = 59 \text{ kJ}$$

which is the heat supplied by the electrical heater.

In summary, we conclude that the 59 kJ of heat supplied slowly by the electrical heater to the gas in side A is used to raise the temperature of the gas in side A to 866 K and for the gas in side A to push the piston outwards by doing 10 kJ of work on the piston. The piston moves without friction to use the 10 kJ of work to adiabatically compress the gas in side B. The work done on the gas on side B raises the temperature of the gas in side B to 553 K.

7.9 Summary

- Differential boundary work is given by

$$dW_{in} = -P\,dV \tag{7.5}$$

  for a quasistatic compression process, and by

$$dW_{out} = P\,dV \tag{7.6}$$

  for a quasistatic expansion process.

- The total boundary work in a quasistatic process is given by

$$W_{out} = -\,W_{in} = \int_{V_o}^{V_f} P\;dV \tag{7.7}$$

- Equations (7.5), (7.6) or (7.7) can be applied to evaluate the boundary work for any substance; solid, liquid, gas or any mixture of phases.
- Boundary work takes the unit J, when P is in Pa and V is in m^3, and it takes the unit kJ, when P is in kPa and V is in m^3.
- Boundary work can also be evaluated using the expression,

$$w_{in} = -\int_{V_o}^{V_f} P\;dv \tag{7.8}$$

  in which v is the specific (or molar) volume, and therefore w_{in} takes the unit kJ/kg (or kJ/kmol), provided P is in kPa, and v is in m^3/kg (or m^3/kmol).

- The boundary work done by a system when executing a process depends on the path followed by the process.

- The area under the curve describing the path of a quasistatic process on a P-V diagram provides the amount of boundary work done by the system if the process is an expansion, and the boundary work done on the system if the process is a compression.

- The area enclosed on a P-V diagram by the path of a quasistatic cyclic process gives the work exchange between the system and its surroundings. This area represents the work done by the system if the process is in a clockwise direction, and the work done on the system if the process is anti-clockwise.

- Boundary work done by a quasistatic constant-pressure process is given by

  $$W_{out} = -\,W_{in} = \int_{V_o}^{V_f} P\,dV = P\,(V_f - V_o) \tag{7.9}$$

  If the system comprises an ideal gas as the working fluid, (7.9) becomes

  $$W_{out} = -\,W_{in} = n\,R\,(T_f - T_o) \tag{7.10}$$

  If the system comprises water/steam as the working fluid, (7.9) becomes

  $$W_{out} = -\,W_{in} = m\,P\,(v_f - v_o)$$

- A constant-volume process does not do any boundary work since the boundary of a system executing a constant-volume process does not expand or contract.

- A quasistatic isothermal process of an ideal gas is described by

  $$P\,V = n\,R\,T = C \tag{7.14}$$

  where C is a constant. Boundary work done on a system undergoing a quasistatic isothermal process is given by

  $$W_{in} = -\,C\,\ln\left(\frac{V_f}{V_o}\right) = -\,C\,\ln\left(\frac{P_o}{P_f}\right). \tag{7.15}$$

  The constant C may be determined using

  $$C = P_o\,V_o = P_f\,V_f = n\,R\,T \tag{7.16}$$

- Boundary work done on a closed system executing an adiabatic process is given by

  $$(W_{in})_{adiabatic} = \Delta U \tag{7.21}$$

  The adiabatic work therefore does not depend on the path of the process, but on the initial and the final states of the process only.

  If the system comprises an ideal gas, (7.21) becomes

  $$(W_{in})_{adiabatic} = n \int_{T_o}^{T_f} C_v \, dT \tag{7.22}$$

  If the system comprises water/steam, (7.21) becomes

  $$(W_{in})_{adiabatic} = m\,(u_f - u_o)$$

- The path of a quasistatic adiabatic process of an ideal gas with constant specific heats is described by

  $$P\,V^{\gamma} = \text{constant} \qquad \text{on a } P\text{-}V \text{ diagram.} \tag{7.29}$$

  $$T\,V^{(\gamma-1)} = \text{constant} \qquad \text{on a } T\text{-}V \text{ diagram.} \tag{7.30}$$

  $$\frac{T}{P^{(\gamma-1)/\gamma}} = \text{constant} \qquad \text{on a } P\text{-}T \text{ diagram.} \tag{7.31}$$

- A quasistatic polytropic process is described by $P\,V^k =$ constant, where k is the polytropic constant. Boundary work done on a system undergoing a polytropic process is given by

  $$W_{in} = \frac{P_f V_f - P_o V_o}{k-1} \qquad \text{for } k \neq 1 \tag{7.34}$$

  For a system comprising an ideal gas, (7.34) becomes

  $$W_{in} = \frac{n\,R\,(T_f - T_o)}{k-1} \qquad \text{for } k \neq 1 \tag{7.35}$$

  For a system comprising water/steam, (7.34) becomes

  $$W_{in} = \frac{m\,(P_f v_f - P_o v_o)}{k-1} \qquad \text{for } k \neq 1$$

- In order to evaluate boundary work using (7.6) to (7.8), the system has to undergo an almost fully resisted expansion or compression. If the expansion (or compression) is not almost fully resisted throughout the entire process then the above equations cannot be used to evaluate boundary work (see Example 7.23).

- It is important to remember that a system expanding into a vacuum does not do any work at all (see Example 7.22).