Chapter 6

WORKING WITH REAL GASES & STEAM

> One's ideas should be as broad as nature if they are to interpret nature.
>
> Sherlock Holmes
> – Sir Arthur Conan Doyle (*A study in Scarlet*)

We have now learned to apply the first law to closed systems containing ideal gas. Ideal gas, which is an imaginary gas used to approximate real gas behaviour, is always in gas phase, but a real gas used in an engineering device may turn into a liquid at appropriate pressures and temperatures. It is therefore necessary that we learn to work with real gases and liquids. In this chapter, we will learn in particular about working with water, which is either in gas phase as steam, or in liquid phase, or in a mixed state of liquid and gas phases.

6.1 Real Gas Behaviour

The intermolecular distances prevailing among the molecules of a real gas at low pressures, are large enough to approximate the behaviour of the real gas by that of the ideal gas. At medium and high pressures, however, the molecules of real gases are brought so close together that the volume occupied by the gas molecules become a considerable fraction of the total gas volume, and the intermolecular forces and their influence on molecular collision processes are no longer negligible. The real gas behaviour therefore deviates significantly from the ideal gas behaviour at high and medium pressures.

Since the ideal gas equation of state cannot be used to satisfactorily describe the behaviour of a real gas, many attempts have been made to modify the ideal gas equation of state to represent the experimental data obtained with real gases. Listed below are few of the many equations available for describing real gas behaviour*. The predictability of these equations is found to be remarkably good when the gas is not too close to the temperature and pressure at which it becomes a liquid.

- van der WAALS EQUATION OF STATE:

Considering the intermolecular attraction forces and the volume occupied by the gas molecules themselves, van der Waals equation of state is written down as

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

where a is the van der Waals constant that takes account of the molecular interactions, and b is the van der Waals constant that compensates for the volume occupied by the molecules. Constants a and b take different values for different substances. Air, for example, has $a = 135.7$ kPa $\cdot$ m^6/kmol^2 and $b = 0.037$ m^3/kmol.

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*For a complete description, see the following sources:

Chapter 2 of Çengel, Y.A. & Boles, M.A. 1998 Thermodynamics: an engineering approach, 3rd Edition, McGraw-Hill International Editions.

Chapter 3 of Rao, Y.V.C. 1997 Chemical Engineering Thermodynamics, Universities Press Limited.

- REDLICH-KWONG (RK) EQUATION OF STATE:

It is one of the widely used equation of state in engineering calculations and is given by

$$P = \frac{RT}{(v-b')} - \frac{a'}{v\,(v+b')\,\sqrt{T}}$$

where a' and b' are constants specific to the substance concerned.

- VIRIAL EQUATION OF STATE:

$$\frac{P\,v}{RT} = 1 + \frac{B}{v} + \frac{C}{v^2} + \frac{D}{v^3} + \cdots$$

where the virial coefficients B, C, D, etc. are functions of temperature alone. Although it is a very complicated procedure to calculate the virial coefficients, once it is done, the virial equation of state predicts the P-v-T behaviour of a real gas with a remarkable accuracy over a wide range.

We have learned in Chapter 5 that, for an ideal gas, u and h are functions of T alone. For real gases, however, u and h depend not only on T, but also on another property such as v or P. It is therefore important to know that changes in u and h for a real gas cannot be evaluated using the methods described for an ideal gas in Chapter 5. However, if the states of the real gas considered is not too close to the temperature and pressure at which it becomes liquid then it is possible to approximate the real gas behaviour to ideal gas behaviour, and use the methods described in Chapter 5 to evaluate the changes in u and h for that real gas.

6.2 Phase Change: Gas to Liquid

A real gas behaviour is such that it could be converted into liquid by decreasing the temperature at appropriate pressures, or by increasing the pressure at appropriate temperatures.

The T-v diagrams of Figure 6.1 shows the phase change behaviour of a pure substance. Water is chosen as the pure substance. Point $\mathbf{A}$ of Figure 6.1(a) represents water at 1 bar and 700°C, at which condition water is in gas phase, and is known as steam. From $\mathbf{A}$ to $\mathbf{B}$, water remains in the form of steam at 1 bar but at different temperatures. The dashed line of Figure 6.1(a) represents the T-v relationship of steam described by the ideal gas equation of state as $T = Pv/R$ at the given pressure. Note that circles representing the real gas behaviour in the region $\mathbf{A}$ to $\mathbf{B}$ of Figure 6.1(a) lie almost on the dashed line of Figure 6.1(a). It means that the ideal gas behaviour of steam adequately describes the real gas behaviour of steam at 1 bar for temperatures above the temperature represented by the point $\mathbf{B}$.

At point $\mathbf{B}$, steam starts to condense, that is, steam begins to convert itself into liquid water. The horizontal line $\mathbf{BD}$ represents the transformation of steam into liquid water at a constant pressure of 1 bar and at a constant temperature of 99.6°C. As steam transforms into liquid water at constant pressure and constant temperature, the specific volume of steam reduces considerably. At point $\mathbf{D}$, all steam is condensed into liquid water.

Note that point $\mathbf{D}$ can also be seen as a point at which water in liquid phase starts its transformation into gas phase, and point $\mathbf{B}$ as a point at which all water in liquid phase completes its transformation into gas phase. It is therefore, the constant temperature represented by the horizontal lines $\mathbf{BD}$ is known as the **boiling point**. The boiling point temperature of a substance is therefore the temperature at which a liquid converts itself into a gas, or a gas converts itself into a liquid, at a given pressure.

From $\mathbf{D}$ to $\mathbf{E}$ of Figure 6.1(a), water remains in liquid form at 1 bar but at different temperatures. Since the specific volume of liquid water experiences very little change with changing temperature, $\mathbf{DE}$ remains almost a vertical line. Behaviour of water in the range of $\mathbf{B}$ to $\mathbf{E}$ cannot be described by ideal gas behaviour since water is no longer in gas phase alone in this region.

The circles of Figure 6.1(b), (c) and (d) represent the actual T-v behaviour of water at 100, 200 and 500 bars, respectively, and the dashed lines represent the behaviour of water predicted by the ideal gas equation. As can clearly be seen in Figure 6.1, the higher is the pressure, the larger is the difference between the actual behaviour and the ideal gas behaviour. It is because the real gas behaviour deviates from ideal gas behaviour at medium and high pressures.

Figure 6.1 The real gas behaviour of a pure substance, represented by circles on a T-v diagram, at (a) = 1 bar, (b) = 100 bar, (c) = 200 bar, and (d) = 500 bar. The dashed line on each of the four plots represents the ideal gas behaviour of the substance, given by $T = Pv/R$. (The numerical values used represent water.)

Figure 6.2 T-v diagram of a pure substance describing the gas phase to liquid phase transformation. (a) T-v relationships at constant pressures, represented by dashed lines. (b) Important characteristics of phase-change. (The numerical values used represent water.)

The boiling point temperature, corresponding to the horizontal line **BD** of Figure 6.1, takes different values at different pressures. It is 99.6°C at 1 bar, 311.0°C at 100 bar, and 365.7°C at 200 bar. The horizontal line **BD** of Figure 6.1, which represents the phase change of water at constant pressure and constant temperature, is present in Figure 6.1(a), (b) and (c), and is not present in Figure 6(d). The reason for this peculiar behaviour of water at 500 bar is discussed at the end of this section.

We have seen that point **B** in Figure 6.1 represents the point at which real gas starts to transform into a liquid. If we take the point **B** at each pressure, and connect them together, we get the curve **CG** shown in Figure 6.2(a). Point **D** in Figure 6.1 represents the point at which the gas just completes its transformation into a liquid. If we take the point **D** at each pressure, and connect them together, we get the curve **LC** shown in Figure 6.2(a). The curve **LCG** is of considerable importance as it separates the area of the T-v diagram into a gas phase region, a liquid phase region, and a liquid-gas phase region, as labelled in Figure 6.2(b). Note that we use the word **vapour** in place of gas in Figure 6.2(b), the reason for which is given at the end of this section.

The important characteristics of the gas to liquid phase change, shown in Figure 6.2(b), are described below:

- Superheated vapour region:

The temperature at any state represented by a point in this region is always **above** the boiling point temperature at the corresponding pressure. A point in this region therefore represents the state of vapour that is **not** about to condense, which means **not** about to become a liquid. It is therefore the vapour in this region is known as the **superheated vapour**, and the region is known as the **superheated vapour region.**

- Saturated vapour line:

The vapour at any state represented by a point along this line (represented by the curve **CG** in Figure 6.2), is at its boiling point temperature. Any heat loss from a vapour at its boiling point temperature, no matter how small, will cause some of the vapour to condense. Since the vapour at a state along the curve **CG** is about to condense, it is known as the **saturated vapour**, and the curve **CG** is known as the **saturated vapour**

line. The pressure at a state along the saturated vapour line is known as the **saturation pressure**. And, the corresponding temperature, which is the boiling point at that pressure, is known as the **saturation temperature**.

- Saturated liquid line:

Any point along this line (represented by the curve **LC** in Figure 6.2), gives the state of liquid that is at its boiling point. Any addition of heat to a liquid at this state, no matter how small, will cause some of the liquid to vapourise. It is therefore the liquid at a state along the curve **LC** is known as the **saturated liquid**, and the curve **LC** is known as the **saturated liquid line**. The pressure of a saturated liquid is the saturation pressure, and its temperature is the saturation temperature.

- Saturated liquid-vapour region:

At any state represented by a single point in this area, what we get is a mixture of saturated liquid and saturated vapour. Therefore, this region is known as the saturated liquid-vapour region, or the wet region. Note that it is only in this region that the vapour and the liquid phases exist together. Along any horizontal line in this region, as any one of the lines **BD** shown in Figure 6.2(a), the temperature and the pressure remain constants at the saturation values. The specific (or molar) volume would, of course, changes from one point to another along any horizontal line in this region.

- Compressed liquid region:

Any point in this region describes the state of liquid that is **not** about to vapourise. That is, the temperature of the compressed liquid is always **below** the boiling point corresponding to the given pressure. Compressed liquid is also known as **subcooled liquid**.

- Critical point:

Point **C** on the curve **LCG** of Figure 6.2 is known as the **critical point**. At the critical point, the saturated vapour phase is the same as the saturated liquid phase, and therefore their properties are identical. The critical point represents the highest pressure and temperature at which the liquid and vapour phases coexist in equilibrium. The respective pressure and temperature at the critical point are known as the **critical pressure**, denoted by P_c, and the **critical temperature**, denoted by T_c.

Different pure substances have different critical pressures and critical temperatures. The critical constants of some selected substances are listed below:

Substance	T_c°C	P_c MPa
Argon	-122.3	4.9
Carbon dioxide	31.0	7.4
Ethane	32.2	4.9
Helium	-268.0	0.23
Hydrogen	-240.0	1.3
Nitrogen	-147.0	3.4
Oxygen	-118.6	5.05
Water	374.15	22.120

The critical temperature and the critical pressure are very crucial properties of a pure substance, and they have many use. For example, the van der Waals constants are defined in terms of P_c and T_c as

$$a = \frac{27\,R^2\,T_c^2}{64\,P_c} \quad \text{and} \quad b = \frac{R\,T_c}{8\,P_c}$$

with R as the gas constant. The constants of Redlich-Kwong equation of state are also defined in terms of P_c and T_c as

$$a' = \frac{0.42748\,R^2\,T_c^{2.5}}{P_c} \quad \text{and} \quad b' = \frac{0.08664\,R\,T_c}{P_c}$$

`Student:` Teacher, I still have no feeling for what a critical point is all about. Would you please tell me more about the critical point?

`Teacher:` Yes, I will. Let us carry out an experiment in a variable volume vessel, in which we hold steam at 500 bar and 700°C, which are above the critical pressure and the critical temperature for water. Now, let us reduce the temperature of steam slowly to room temperature while holding the pressure at 500 bar. The path of this experiment can be seen in the T-v diagram of Figure 6.1(d). The path, as seen in this diagram, has no horizontal line at constant temperature, along which steam transforms into liquid water.

`Student:` Does that mean that steam does not become liquid water if the pressure is held above the critical pressure?

`Teacher:` No, it does not mean that. Even if the pressure is larger than the critical pressure, if the temperature is reduced to a low enough value then what we will have is liquid water. You could see in Figure 6.2(a) that, at low temperatures, the path of water at 500 bar is nearly the same as the paths at other pressures which are lower than the critical pressure. All these paths lie almost on the saturated liquid line **LC**.

`Student:` What exactly is the difference then?

`Teacher:` The difference is, for pressures below the critical pressure, we know exactly when the gas phase transforms into the liquid phase. We could see the transformation occurring at a constant temperature, with our own eyes. For pressures above the critical pressure, on the other hand, we will not be able to tell exactly when the gas phase transformed into the liquid phase. It is because this transformation occurs gradually.

`Student:` I think I am beginning to see what a critical point is all about. Teacher, would you please tell me what is the difference between a vapour and a gas?

`Teacher:` Vapour is a word used to describe a substance in gas phase at temperatures below critical temperature, and gas is used for the same at temperatures above critical temperature. Except for this difference, the gas and vapour states are identical to each other.

`Student:` I got it. The critical temperature of water is 374.15°C. That is why we refer to water in gaseous phase at room temperature, or even at 100°C, as vapour. Whereas, we refer to oxygen at room temperature as gas since its critical temperature is -118.6°C.

`Teacher:` Yes, that is correct.

6.3 P-v diagram for Phase Change

The P-v behaviour of water at four different temperatures are shown in Figure 6.3(a), and its characteristics are labeled on Figure 6.3(b).

Figure 6.3 P-v diagram of a pure substance describing the gas phase to liquid phase transformation. (a) P-v relationships at constant temperatures, represented by dashed lines. (b) Important characteristics of phase-change. (The numerical values used represent water.)

Let us take a look at the path of water at 300°C. Point **E** in Figure 6.3(a) represents the state of liquid water at 350 bar and 300°C. When the pressure is reduced, while maintaining the temperature constant at 300°C, the specific volume of liquid water increases slightly until point **D** is reached. At this point, the pressure is 85.93 bar.

Any further reduction in pressure is not possible until all the liquid water is transformed into water vapour at 300°C. During the vaporization process, which is described by the path **DB**, the specific volume increases considerably at constant pressure and temperature. At point **B** all the liquid water is transformed into water vapour. Any further reduction in pressure beyond point **B** at 300°C results in considerable increase in the specific volume of water vapour, as shown in Figure 6.3(a).

Similar behaviour is repeated by water at 100°C, as well. For water at 400°C and at 500°C, which are temperatures above the critical temperature 374.15°C, the transformation of water from liquid phase to gas phase is so gradual, as shown in Figure 6.3(a), that it is not possible to tell exactly when the liquid water turns into water vapour. By the way, as we have already seen, the gaseous phase of water at temperatures above critical temperature is referred to as gas, not as vapour, even though the gas and vapour states are identical to each other.

The curve **LCG** shown in Figure 6.3(a) separates the area on the P-v diagram into a vapour phase region, a liquid phase region, and a liquid-vapour phase region. These regions are labelled in Figure 6.3(b). The T-v diagrams of Figure 6.2 and the P-v diagrams of Figure 6.3 are very similar to each other except for the orientation of the constant pressure lines of the T-v diagrams in comparison with the orientation of the constant temperature lines of the P-v diagrams.

6.4 P-T diagram for Phase Change

The saturated liquid-vapour regions on the respective T-v and P-v diagrams of Figure 6.2(b) and Figure 6.3(b), reduce to a curve on a P-T diagram, since the liquid to vapour, or the vapour to liquid, phase change occurs at a constant temperature for a chosen pressure. This curve is known as the **vaporization curve**, and is shown in Figure 6.4. Each point along this curve gives the pressure-temperature set at which the liquid and vapour phases remain in equilibrium.

One end of the vaporization curve is the critical point, which represents the highest pressure and temperature at which the liquid and vapour phases coexist in equilibrium. The other end of the vaporization curve is the **triple point**, which represents the state at which it is possible to maintain an equilibrium mixture of all three phases, solid, liquid and vapour. For water, the triple point is at a pressure of 0.006112 bar and a temperature of 0.01°C.

Figure 6.4 also shows the **melting curve** (or fusion curve), along which the solid and liquid phases are in equilibrium. The **sublimation curve** of Figure 6.4 represents the pressure-temperature sets at which a direct phase change between the solid phase and the gas phase exists. Note that the fusion, vaporization and sublimation curves of the P-T diagram meet at the triple point.

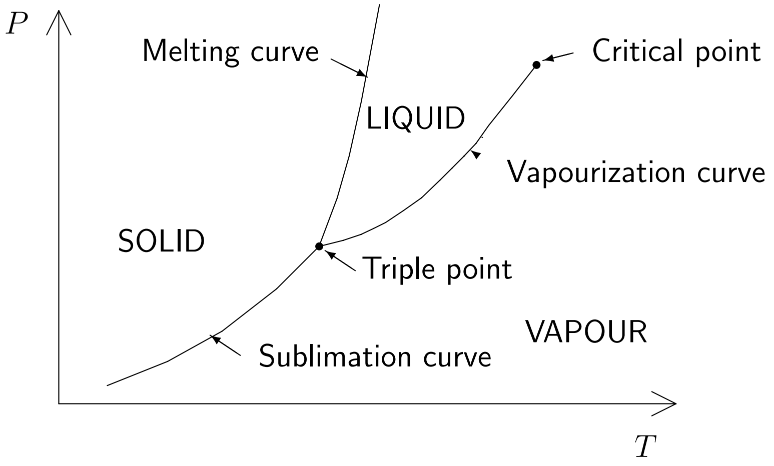

Figure 6.4 P-T diagram for phase changes (not drawn to scale).

6.5 The Steam Table

Ideal gas relationships developed in Chapter 5 can be used to describe the real gas behaviour when the state of the real gas is not too close to either the critical point or the saturated vapour line. In many real life situations, however, a more accurate representation of the thermodynamic properties than the use of ideal gas relationships is demanded. In such cases, thermodynamics properties are obtained from **thermodynamic property tables.**

Thermodynamic property tables contain data derived from a combination of extensive and precise experiments and calculations. Let us here concentrate on the thermodynamic properties of liquid water and steam, which are tabulated in a table known as the **Steam Table.** There are many versions of the steam table. Each version differ from the other in some aspects, but all of them have a common basic structure.

A Steam Table is usually divided into three parts, which are Saturated Water and Steam Table, Superheated Steam Table and Compressed Water Table. A detail account of how the data are presented in these three parts of the Steam Table, and the manner in which the data could be extracted from the table are discussed below.

6.5.1 Saturated Water and Steam Table

Saturated Water and Steam Table, also referred to as Saturated Steam Table, lists the data corresponding to point B and point D shown in Figure 6.5. Point B represents the saturated steam state that lies on the saturated vapour line, represented by the curve **CG** of Figure 6.2(b) or Figure 6.3(b). Point D represents the saturated water state that lies on the saturated liquid line, represented by the curve **LC** of Figure 6.2(b) or Figure 6.3(b).

The pressure of saturated water or steam is the saturation pressure, denoted by P_{sat} or simply by P_s. And, the corresponding temperature is the saturation temperature, denoted by T_{sat} or simply by T_s. (It is also common to use the symbols p_s and t_s for saturation pressure and saturation temperature, respectively.) The saturation pressure and temperature are related to each other, and this relationship is represented by the vaporization curve shown in Figure 6.4. In the saturated water and steam region, therefore, either the pressure or temperature can be chosen with complete freedom, not both.

In some Saturated Water and Steam Tables, temperature is chosen as the independent property, and is entered as the first entry in the table in regular steps. And, all the other properties of saturated steam and water that depend on the temperature are listed against the corresponding temperature entry. In some other tables, pressure is chosen as the independent property, and is entered as the first entry in regular steps, and the other properties are listed against it. Many tables contains both.

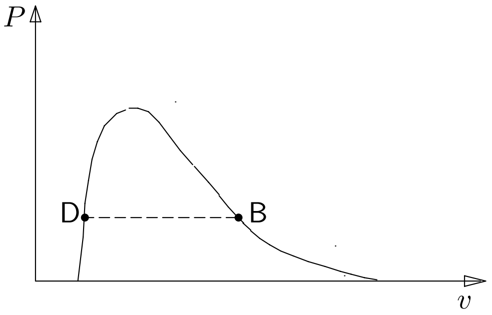

Figure 6.5 Points B and D represent typical entries of the saturated steam state and the saturated liquid water state, respectively, on the Saturated Water and Steam Table.

The entries in the Saturated Water and Steam Table begin with the pressure of 0.006112 bar and 0.01°C, which is the triple point, and ends with the pressure of 221.2 bar and 374.15°C, which is the critical point. The properties of saturated vapour, other than the pressure and temperature, are identified by the subscript $_g$ in the steam table. Specific volume of the saturated vapour is denoted by v_g, specific internal energy by u_g, specific enthalpy by h_g, and specific entropy by s_g. The properties of saturated liquid are identified by the subscript $_f$ in the steam table. Specific volume of the saturated liquid is denoted by v_f, specific internal energy by u_f, specific enthalpy by h_f, and specific entropy by s_f. Some Saturated Water and Steam Tables also contains entries for u_{fg}, h_{fg} and s_{fg}, where $u_{fg} = u_g - u_f$, $h_{fg} = h_g - h_f$ and $s_{fg} = s_g - s_f$.

`Student:` Teacher, what is specific entropy?

`Teacher:` Well, entropy divided by the mass of the substance gives the specific entropy.

`Student:` What is entropy, then?

`Teacher:` Oh...Yes. You have not been formally introduced to entropy yet. Chapter 11 is dedicated to entropy, where you will be formally introduced to entropy. You will learn to work out problems using entropy in that chapter. Now, let me tell you that entropy is a property just as pressure, temperature, internal energy and enthalpy are. And, we will learn here how to refer to the property specific entropy from a Steam Table in the same way as we learn to refer to other specific properties from the Steam Table. Is that okay?

`Student:` Yes, it's okay.

Example 6.1

Determine the specific volume, specific internal energy, specific enthalpy and the specific entropy of saturated water vapour at 100 kPa.

Solution to Example 6.1

A typical entry for saturated steam in a Saturated Water and Steam Table at 100 kPa (= 1 bar) is shown below where the subscript $_g$ refers to properties of saturated vapour.

p bar	t_s °C	v_g m^3/kg	u_f -	u_g kJ/kg	h_f -	h_{fg} -	h_g kJ/kg	s_f -	s_{fg} -	s_g kJ/kg · K
0.90	96.7	1.869	-	2502	-	-	2671	-	-	7.394
0.95	98.2	1.777	-	2504	-	-	2673	-	-	7.376
1.00	99.6	1.694	-	2506	-	-	2675	-	-	7.359
1.10	102.3	1.549	-	2510	-	-	2680	-	-	7.327
1.20	104.8	1.428	-	2512	-	-	2683	-	-	7.298

The specific volume of the saturated vapour at 1 bar is given by the third entry in the third row, the specific internal energy by the fifth entry, the specific enthalpy by the eighth entry, and the specific entropy by the eleventh entry. Taking the units used in that particular table, we can write v_g = 1.694 m^3/kg, u_g = 2506 kJ/kg, h_g = 2675 kJ/kg and s_g = 7.359 kJ/kg · K. The second entry yields the saturation temperature. That is, the saturated vapour is at 99.6°C.

Example 6.2

Determine the specific volume, specific internal energy, specific enthalpy and the specific entropy of saturated liquid water at 100 kPa.

Solution to Example 6.2

A typical entry for saturated water in a Saturated Water and Steam Table at 100 kPa (= 1 bar) is shown below where the subscript $_f$ refers to the properties of saturated liquid.

p bar	t_s °C	v_g -	u_f kJ/kg	u_g -	h_f kJ/kg	h_{fg} -	h_g -	s_f kJ/kg · K	s_{fg} -	s_g -
0.90	96.7	1.869	405	-	405	-	-	1.270	-	-
0.95	98.2	1.777	411	-	411	-	-	1.287	-	-
1.00	99.6	1.694	417	-	417	-	-	1.303	-	-
1.10	102.3	1.549	429	-	429	-	-	1.333	-	-
1.20	104.8	1.428	439	-	439	-	-	1.361	-	-

The fourth entry in the third row gives the specific internal energy, the sixth entry the specific enthalpy and the ninth entry the specific entropy. Therefore, u_f = 417 kJ/kg, h_f = 417 kJ/kg and s_f = 1.303 kJ/kg · K. The saturated liquid is at 99.6°C.

The entry for specific volume of the saturated liquid is not available on the table above. Therefore, make use of $h = u + pv$ to calculate the specific volume of the saturated liquid as $v_f = (h_f - u_f)/p = 0$. This impossible result is a consequence of the manner in which the entries for u and h are rounded off in the Saturated Water and Steam Table concerned.

For a realistic value of v_f, therefore, we must search for steam tables containing tabulated values of v_f. Such entry found in a Saturated Water and Steam Table[†] is reproduced below:

t °C	p_s bar	$v_f \times 10^2$ m^3/kg	C_{pf} kJ/kg · K	C_{pg} kJ/kg · K	- -	- -	- -	- -	- -	- -
90	0.7011	0.1036	4.208	1.97	-	-	-	-	-	-
95	0.8453	0.1040	4.213	1.99	-	-	-	-	-	-
100	1.01325	0.1044	4.219	2.01	-	-	-	-	-	-
105	1.208	0.1048	4.226	2.03	-	-	-	-	-	-

[†] See, for example, page 10 of Mayhew, Y.R. and Rogers, G.F.C. 1972 Thermodynamics and Transport Properties of Fluids, SI Units, 2^{nd} Edition, Oxford, Basil Blackwell.

As you can see, the entry for 1 bar pressure is not available in the above tabulation, but entries for 0.8453 bar and 1.01325 bar are available. A linear interpolation between the entries corresponding to 0.8453 bar and 1.01325 bar would yield the specific volume required at 1 bar as follows:

$$\frac{v_f \times 10^2 - 0.1040}{0.1044 - 0.1040} = \frac{1 - 0.8453}{1.01325 - 0.8453}$$

which gives $v_f = 0.10437 \times 10^{-2}$ m^3/kg at 1 bar. This volume can be approximated to 0.001 m^3/kg.

That means v of saturated liquid at 1 bar $\approx$ (1/1000) m^3/kg = (1/density of water). At pressures that are not so high, it is a general practice to take the volume of saturated and subcooled (compressed) waters to be (1/1000) m^3/kg. That is why most of the steam tables do not carry the values of v_f at low and moderate pressures.

6.5.2 Dryness Fraction

In the previous section, we have learned to determine the properties of saturated steam and saturated water from the Saturated Water and Steam Table. That is to say, we have learned to determine the properties at any state along the curve **LCG** of Figure 6.2(b) or Figure 6.3(b). In this section, we will learn to determine the property of a state represented by any point in the saturated liquid-vapour region bounded by the curve **LCG**.

One such state is represented by point F in Figure 6.6. At this state, we know that a pure substance exists as part liquid and part vapour. That is, it is a mixture of saturated liquid and saturated vapour. Under ideal mixing conditions, the properties of the saturated liquid and the saturated vapour in this mixture are represented by points D and B, respectively.

To analyze such a mixture properly, we need to know the portions of the saturated liquid and saturated vapour phases present in the mixture. This is done by introducing a quantity known as **dryness fraction**, denoted by x. Dryness fraction, which is also known as **quality**, is defined as the mass fraction of saturated vapour in a saturated vapour-liquid mixture. That is,

$$x = \frac{m_g}{m_T} \tag{6.1}$$

with

$$m_T = m_g + m_f \tag{6.2}$$

where m_f, m_g and m_T are the respective masses of the saturated liquid, saturated vapour and the saturated vapour-liquid mixture.

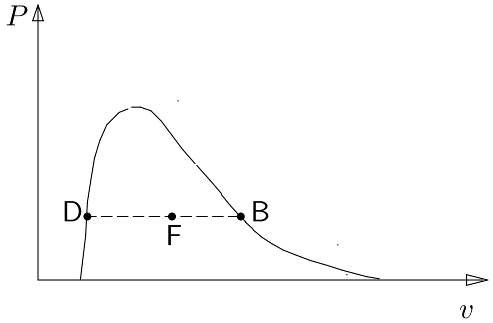

Figure 6.6 Point F represents a typical saturated liquid-vapour mixture.

If the given mixture lies on the saturated liquid line, as in the case of point D, then it contains no vapour. Therefore, $x = 0$. If the given mixture lies on the saturated vapour line, as in the case of point B, then it contains only vapour. Therefore, $x = 1$. For any mixture that lies in between these two extremes, as in the case of point F, x takes a value between 0 and 1. Note that dryness fraction has significance only for saturated mixtures.

Since a saturated mixture can be treated as a combination of saturated liquid and saturated vapour, the property such as the specific volume of a saturated mixture can be found as follows:

$$v = \frac{V_{Total}}{m_T} = \frac{V_f + V_g}{m_T} = \frac{m_f * v_f + m_g * v_g}{m_T} \tag{6.3}$$

where v_g and v_f are the respective specific volumes of the saturated vapour and the saturated liquid in the mixture.

Combining (6.1), (6.2) and (6.3) to eliminate the masses, we get

$$v = (1-x)\,v_f + x\,v_g \; = \; v_f + x\,(v_g - v_f) \; = \; v_f + x\,v_{fg} \tag{6.4}$$

where $v_{fg} \equiv v_g - v_f$. The expression for v in (6.4) can be interpreted as follows. The mixture has at least the volume of the saturated liquid, but has in addition the volume gained by vaporization of the fraction represented by x.

The analysis given above can be repeated for specific internal energy, specific enthalpy and specific entropy of the saturated mixture with the following results:

$$u = u_f + x\,(u_g - u_f) = u_f + x\,u_{fg} \tag{6.5}$$
$$h = h_f + x\,(h_g - h_f) = h_f + x\,h_{fg} \tag{6.6}$$
$$s = s_f + x\,(s_g - s_f) = s_f + x\,s_{fg} \tag{6.7}$$

Another quantity that is used to represent the portions of the liquid and vapour phases in the mixture is **moisture**, which is defined as

$$\text{moisture} = \frac{\text{mass of saturated liquid}}{\text{total mass of the mixture}} = \frac{m_f}{m_T} \tag{6.8}$$

Therefore, moisture can be related to dryness fraction (or quality) as follows:

$$\text{moisture} = \frac{m_T - m_g}{m_T} = 1 - x = 1 - \text{dryness fraction} \tag{6.9}$$

Example 6.3

Determine v, u and h of water at 150 kPa and 10% dryness fraction (or quality).

Solution to Example 6.3

It is a mixture of saturated steam and water at 150 kPa with the dryness fraction of $x = 0.10$. Therefore, (6.4), (6.5) and (6.6) must be used to determine v, u and h, respectively. A typical entry in a Saturated Water and Steam Table at 150 kPa (= 1.5 bar) is shown below:

p bar	t_s °C	v_g m^3/kg	u_f kJ/kg	u_g kJ/kg	h_f kJ/kg	h_{fg} kJ/kg	h_g kJ/kg	-	-	-
1.4	109.3	1.236	458	2517	458	2232	2690	-	-	-
1.5	111.4	1.159	467	2519	467	2226	2693	-	-	-
1.6	113.3	1.091	475	2521	475	2221	2696	-	-	-

From the third entry in the second row of the table above we get $v_g = 1.159$ m^3/kg. The value of v_f may be evaluated in a method similar to the one discussed in the **Solution to Example 6.2**, or may be approximated to 0.001 m^3/kg. Then

$$v = 0.001 + 0.1 \times (1.159 - 0.001) = 0.117 \text{ m}^3/\text{kg}$$

We can calculate u and h using similar procedure as follows:

$$\begin{aligned} u &= u_f + x\,(u_g - u_f) = 467 + 0.1 \times (2519 - 467) = 672.2 \text{ kJ/kg} \\ h &= h_f + x\,h_{fg} = 467 + 0.1 \times 2226 = 689.6 \text{ kJ/kg} \end{aligned}$$

The mixture is at its saturated temperature of 111.4°C.

6.5.3 Superheated Steam Table

Superheated Steam Table lists the data corresponding to any state represented by a point in the superheated vapour region of Figure 6.2(b) or Figure 6.3(b). One such state is represented by point A of Figure 6.7. In the superheated vapour region, two properties can be chosen with complete freedom. Pressure and temperature are used as the two independent properties in the Superheated Steam Table.

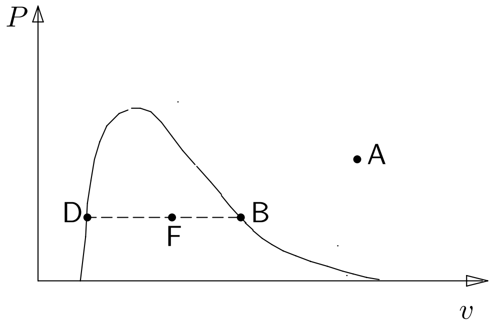

Figure 6.7 Point A represents a typical entry of the superheated vapour state on the Superheated Steam Table.

Example 6.4

Determine v, u, h and s of water at 4 MPa and 350°C.

Solution to Example 6.4

First of all, we must find out if the given state is a saturated state or a superheated state or a compressed state. A typical entry corresponding to the pressure of 4 MPa (or 40 bar) found from a Saturated Water and Steam Table is reproduced below:

p bar	t_s °C	v_g m^3/kg	u_f kJ/kg	u_g kJ/kg	h_f -	h_{fg} -	h_g -	s_f -	s_{fg} -	s_g -
40	250.3	0.04977	1082	2602	-	-	-	-	-	-

The saturated temperature at 40 bar is 250.3°C. This means that water boils at 250.3°C at 40 bar. Therefore, steam at 350°C at 40 bar is at the superheated state. Typical entries corresponding to superheated steam at 40 bar in a Superheated Steam Table are reproduced below:

p (t_s)			t	200	250	300	350	400	-
	v_g	0.0498	v			0.0588	0.0664	-	-
40	u_g	2602	u			2728	2828	-	-
(250.3)	h_g	2801	h			2963	3094	-	-
	s_g	6.070	s			6.364	6.584	-	-

where the units used for p, t, v, u, h and s are bar, °C, m^3/kg, kJ/kg, kJ/kg and kJ/kg·K, respectively.

The first column of the entry shows the pressure 40 bar and the corresponding saturated temperature 250.3°C given within the brackets. The second column gives the saturated vapour properties as $v_g = 0.0498$ m^3/kg, $u_g = 2602$ kJ/kg, $h_g = 2801$ kJ/kg and $s_g = 6.070$ kJ/kg·K.

The rest of columns shows the superheated steam properties in the order v, u, h and s at 200°C, 250°C, 300°C, 350°C, etc. at 40 bar pressure. Note that the columns corresponding to the temperatures of 200°C and 250°C are empty since these temperatures are below the saturated temperature 250.3°C.

The entry corresponding to 350°C would give the properties of superheated steam at 40 bar and 350°C as $v = 0.0664$ m^3/kg, $u = 2828$ kJ/kg, $h = 3094$ kJ/kg and $s = 6.584$ kJ/kg·K.

Example 6.5

Determine v, u, h and s of water at 100 bar and 320°C.

Solution to Example 6.5

It can be seen from any Saturated Water and Steam Table that the saturated temperature at 100 bar is 311.0°C. Therefore, steam at 100 bar and 320°C is at superheated state and the corresponding entry found in a Superheated Steam Table is shown below:

p (t_s)			t	350	375
100	v_g	0.01802	$v\times 10^2$	2.241	2.453
(311.0)	h_g	2725	h	2926	3017
	s_g	5.615	s	5.947	6.091

where the units used for p, t, v, h and s are bar, °C, m^3/kg, kJ/kg and kJ/kg·K, respectively.

Superheated steam properties at the required temperature of 320°C are not available in the entries shown above. Therefore, we must use the method of linear interpolation between the saturated steam entries at 311°C and the superheated steam entries at 350°C to obtain v, h and s at 100 bar and 320°C as follows:

$$\frac{v - 0.01802}{0.02241 - 0.01802} = \frac{320 - 311}{350 - 311}$$

gives $v = 1.903\times 10^{-2}$ m^3/kg. Similarly,

$$\frac{h - 2725}{2926 - 2725} = \frac{s - 5.615}{5.947 - 5.615} = \frac{320 - 311}{350 - 311}$$

gives $h = 2771$ kJ/kg and $s = 5.692$ kJ/kg·K.

Since the entry for u is not found at the given state, let us use the following procedure to determine u:

$$\begin{aligned} u &= h - pv \\ &= (2771 \text{ kJ/kg}) - (100 \text{ bar})(1.903 \times 10^{-2} \text{ m}^3/\text{kg}) = 2581 \text{ kJ/kg} \end{aligned}$$

Therefore, for superheated steam at 100 bar and 320°C, $v \approx 1.903 \times 10^{-2}$ m^3/kg, $u \approx 2581$ kJ/kg, $h \approx 2771$ kJ/kg and $s \approx 5.692$ kJ/kg·K. The approximate signs are introduced to show that use of linear interpolation may give approximate results only. We have assumed that the properties of steam change linearly with temperature between 311°C and 350°C at 100 bar, which is not necessarily true, and therefore the results obtained may only be approximate.

Example 6.6

Determine v, u, h and s of water at 430 bar and 680°C.

Solution to Example 6.6

The given pressure and temperature are well above the critical pressure and temperature for water, and therefore the given state is a superheated state. The corresponding entries found in a steam table are reproduced below:

p	t	-	-	600	700	-
400	$v\times 10^2$	-	-	0.809	0.993	-
	h	-	-	3348	3677	-
	s	-	-	6.014	6.371	
450	$v\times 10^2$	-	-	0.698	0.870	-
	h	-	-	3299	3644	-
	s	-	-	5.914	6.290	

The units used for p, t_s, v, u, h and s are bar, °C, m^3/kg, kJ/kg, kJ/kg and kJ/kg$\cdot$K, respectively.

We have to first interpolate between the pressures 400 bar and 450 bar to obtain the corresponding entries at 430 bar pressure. Then we have to interpolate between 600°C and 700°C to get the results required at 680°C. Interpolating between the pressures 400 bar and 450 bar entries corresponding to 600°C as

$$\frac{v\times 10^2 - 0.809}{0.698 - 0.809} = \frac{h - 3348}{3299 - 3348} = \frac{s - 6.014}{5.914 - 6.014} = \frac{430 - 400}{450 - 400}$$

gives $v = 0.742\times10^{-2}$ m^3/kg, $h = 3319$ kJ/kg and $s = 5.954$ kJ/kg$\cdot$K at 600°C. Interpolating between the pressures 400 bar and 450 bar entries corresponding to 700°C as

$$\frac{v\times 10^2 - 0.993}{0.870 - 0.993} = \frac{h - 3677}{3644 - 3677} = \frac{s - 6.371}{6.290 - 6.371} = \frac{430 - 400}{450 - 400}$$

gives $v = 0.919\times10^{-2}$ m^3/kg, $h = 3657$ kJ/kg and $s = 6.322$ kJ/kg$\cdot$K at 430 bar and 700°C.

The results can be tabulated as follows:

p	t	-	-	600	700	-
430	$v\times 10^2$	-	-	0.742	0.919	-
	h	-	-	3319	3657	-
	s	-	-	5.954	6.322	

Interpolating between 600°C and 700°C to get the values for 680°C as

$$\frac{v \times 10^2 - 0.742}{0.919 - 0.742} = \frac{h - 3319}{3657 - 3319} = \frac{s - 5.954}{6.322 - 5.954} = \frac{680 - 600}{700 - 600}$$

gives $v = 0.884 \times 10^{-2}$ m^3/kg, $h = 3589$ kJ/kg, $s = 6.248$ kJ/kg·K at 680°C and 430 bar.

To get u, let us use

$$\begin{aligned} u &= h - pv \\ &= (3589 \text{ kJ/kg}) - (430 \text{ bar})(0.884 \times 10^{-2} \text{ m}^3\text{/kg}) = 3209 \text{ kJ/kg} \end{aligned}$$

Therefore, for superheated steam at 430 bar and 680°C, $v \approx 0.884 \times 10^{-2}$ m^3/kg, $u \approx 3209$ kJ/kg, $h \approx 3589$ kJ/kg and $s \approx 6.248$ kJ/kg·K. The approximate signs are introduced to show that use of linear interpolation gives only approximate results.

6.5.4 Compressed Water Table

Compressed Water Table lists the data corresponding to any state represented by a point in the compressed water region of Figure 6.2(b) or Figure 6.3(b). One such state is represented by point E of Figure 6.8. In the compressed water region, two properties can be chosen with complete freedom.

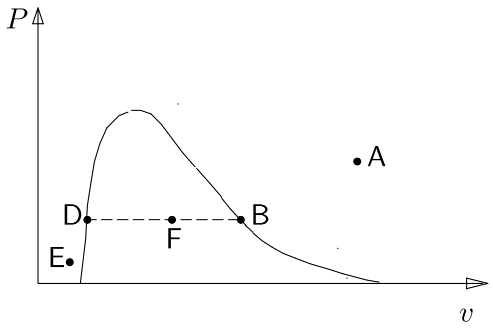

Figure 6.8 Point E represents a typical compressed water state on a Compressed Water Table.

Example 6.7

Determine v, u, h and s of water at 7.5 MPa and 100°C.

Solution to Example 6.7

It can be seen from any Saturated Water and Steam Table that the saturated temperature at 7.5 MPa (= 75 bar) is 290.5°C. This means that water boils at 290.5°C at 75 bar. Therefore at the given temperature of 100°C, the water is at subcooled (or compressed) state. Typical entries for compressed water at 100°C found in a Compressed Water Table[‡] at 50 bar and 100 bar are reproduced below:

P bar	v m^3/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
50	0.0010410	417.52	422.72	1.3030
100	0.0010385	416.12	426.50	1.2992

The pressure of interest is 75 bar and the above entries are at 50 bar and 100 bar, and therefore we use the method of linear interpolation to get the values at the pressure specified as follows:

$$\frac{v - 0.0010385}{0.0010410 - 0.0010385} = \frac{75 - 100}{50 - 100}$$

gives $v \approx 0.0010398$ m^3/kg at 75 bar and 100°C. Similarly,

$$\frac{u - 416.12}{417.52 - 416.12} = \frac{h - 426.50}{422.72 - 426.50} = \frac{s - 1.2992}{1.3030 - 1.2992} = \frac{75 - 100}{50 - 100}$$

gives $u \approx 416.82$ kJ/kg, $h \approx 424.61$ kJ/kg and $s \approx 1.3011$ kJ/kg · K at 75 bar and 100°C.

Example 6.8

Determine v, u, h and s of water at 6 bar and 50°C.

[‡] Table A-7 of Çengel, Y.A. & Boles, M.A. 1998 Thermodynamics: an engineering approach, 3^{rd} Edition, McGraw-Hill International Editions.

Solution to Example 6.8

It can be seen from any Saturated Water and Steam Table that the saturated temperature at 6 bar is 158.8°C. Therefore, 6 bar and 50°C represent a compressed water state, and the properties at this state are to be found.

Entries in many Compressed Water Tables, however, start at about 50 bar since compressed water properties are relatively independent of the pressure at relatively low pressures such as 6 bar. It is therefore we shall use the general approximation, that is **"to treat compressed liquid as saturated liquid at the given temperature"**. This is because the compressed water (or liquid) properties depend on temperature more strongly than they do on pressure.

Thus, in the absence of compressed liquid data, it can be assumed that the respective values of v, u, h and s required at a given pressure and temperature are approximately equal to v_f, u_f, h_f and s_f at the given temperature, which are the properties of the saturated liquid at that temperature.

The saturated liquid at 50°C, as we can find from a a Saturated Water and Steam Table, is at 0.1233 bar. At that state, $v_f = 0.1012 \times 10^{-2}$ m^3/kg, $h_f = 209.3$ kJ/kg, and $s_f = 0.704$ kJ/kg·K. The specific internal energy could be worked out using

$$u_f = h_f - P\,v_f = 209.3 - (0.1233 \times 10^2)(0.1012 \times 10^{-2}) = 209.29 \text{ kJ/kg}$$

Therefore, the properties at 6 bar and 50°C may be given as $v \approx 0.1012 \times 10^{-2}$ m^3/kg, $u \approx 209.29$ kJ/kg, $h \approx 209.3$ kJ/kg and $s \approx 0.704$ kJ/kg·K.

Example 6.9

Determine h of water at 6 bar and 50°C using C_p for saturated liquid water as 4.182 kJ/kg·K at 50°C and 4.35 kJ/kg·K at 160°C.

Solution to Example 6.9

Let us imagine that we heat water from the subcooled water state at 6 bar and 50°C to the saturated water state at 6 bar and 158.8°C at a constant pressure of 6 bar. For a constant-pressure process of water, we can apply (5.13) as

$$\Delta h = h_f - h_{50^oC} = \int_{50^oC}^{t_s} C_p\, dT$$

which is applicable for water since it is in one single phase during the heating process from 50°C to the saturation temperature t_s = 158.8°C at 6 bar. The saturated liquid enthalpy at 6 bar is h_f = 670 kJ/kg. Therefore

$$670 - h_{50^oC} = \int_{50+273}^{158.8+273} C_p\, dT$$

The specific heat at constant volume C_p changes with temperature and pressure. We can ignore the dependence of C_p on pressure. We are given that C_p for saturated liquid water is 4.182 kJ/kg·K at 50°C and 4.35 kJ/kg·K at about 160°C. Taking an average value for C_p as (4.182+4.35)/2, we can calculate h as

$$h_{50^oC} = 670 - \frac{(4.182 + 4.35)}{2}(158.8 - 50) = 205.9 \text{ kJ/kg}$$

Comment: Note that the **Solution to Example 6.8** and the **Solution to Example 6.9** exhibit two different ways of finding an approximate value of the enthalpy of compressed water at pressures for which the compressed water properties are not readily available in Compressed Water Tables.

6.6 Worked Examples

Example 6.10

Methane gas stored in a 10 m^3 rigid tank is at 2.0 MPa pressure and 300 K temperature. Owing to undetected leakage, the pressure in the tank is reduced to 1 bar while the temperature remained at 300 K. Determine the mass of methane lost due to leakage, assuming ideal gas behaviour.

Solution to Example 6.10

Amount of methane lost due to the leakage can be found if we know the amounts of methane in the tank before and after the development of the leak.

Before the leak developed, methane in the tank is at 2.0 MPa pressure and 300 K temperature, and the volume of the tank is 10 m^3. Therefore, the amount of methane in the tank before the leak, denoted by n_o, can be calculated using the ideal gas equation of state as follows:

$$n_o = \frac{(2.0\ \text{MPa} \times 10^3\ \text{kPa/MPa})\ (10\ \text{m}^3)}{(8.314\ \text{kJ/kmol}\cdot\text{K})\ (300\ \text{K})}\ =\ 8.02\ \text{kmol}$$

from which the mass of methane before the leak is found as

$$m_o = (8.02\ \text{kmol}) \times (16\ \text{kg/kmol}) = 128.3\ \text{kg}$$

After the leak, the pressure is 1 bar and the temperature is 300 K. The volume of the tank remains the same at 10 m^3 since the tank is rigid. The amount of methane in the tank after the leak, denoted by n_f, can be calculated using the ideal gas equation of state as follows:

$$n_f = \frac{(1\ \text{bar} \times 10^2\ \text{kPa/bar})\ (10\ \text{m}^3)}{(8.314\ \text{kJ/kmol}\cdot\text{K})\ (300\ \text{K})}\ =\ 0.40\ \text{kmol}$$

from which the mass of methane before the leak is found as

$$m_f = (0.40\ \text{kmol}) \times (16\ \text{kg/kmol}) = 6.4\ \text{kg}$$

The mass of methane lost due the leakage is therefore

$$= m_o - m_f = 128.3\ \ \text{kg} - 6.4\ \ \text{kg} = 121.9\ \text{kg}.$$

Example 6.11

Redo **Example 6.10** considering the real gas behaviour of methane.

Solution to Example 6.11

Before the leak, methane is at 2.0 MPa and 300 K and it occupies 10 m^3 volume. Table A.7.2SI of Van Wylan, Sonntag & Borgnakke[§] lists the real gas

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[§] Van Wylan, G., Sonntag, R. & Borgnakke, C. 1994 *Fundamentals of Classical Thermodynamics*, 4th Edition, John Wiely & Sons.

properties of methane. From which we get the specific volume of methane at 2.0 MPa and 300 K as 0.0751 m^3/kg. The mass of methane before the leak is therefore given by

$$m_o = \frac{\text{total volume}}{\text{specific volume}} = \frac{10\ \mathrm{m}^3}{0.0751\ \mathrm{m}^3/\mathrm{kg}} = 133.2\ \mathrm{kg}$$

After the leak, methane is at 1 bar and 300 K and it still occupies 10 m^3 volume. Table A.7.2SI of Van Wylan, Sonntag & Borgnakke lists the specific volume of methane at 1 bar, which is 0.1 MPa, and 300 K as 1.5521 m^3/kg. The mass of methane after the leak is therefore given by

$$m_f = \frac{\text{total volume}}{\text{specific volume}} = \frac{10\ \mathrm{m}^3}{1.5521\ \mathrm{m}^3/\mathrm{kg}} = 6.44\ \mathrm{kg}$$

The mass of methane lost due the leakage is therefore

$$= m_o - m_f = 133.2\ \ \mathrm{kg} - 6.44\ \ \mathrm{kg} = 126.8\ \mathrm{kg}.$$

Note that the mass of methane **after the leak**, calculated assuming ideal gas behaviour (**Example 6.10**) is nearly the same as that calculated considering the real gas behaviour of methane (**Example 6.11**). It is because methane is at 1 bar after the leak, and 1 bar can be considered as low pressure. At low pressures, real gases imitate ideal gas behaviour. Thus, calculating the mass of methane at 1 bar pressure assuming ideal gas behaviour does not introduce any significant error in the calculation.

The mass of methane **before the leak** is 128.3 kg when ideal gas behaviour is assumed (**Example 6.10**), and 133.2 kg when real gas behaviour is considered (**Example 6.11**). It is because methane is at 2.0 MPa before the leak, which is nearly 20 times the atmospheric pressure, and therefore does not fall in the category of low pressure. Calculating the mass of methane at 2.0 MPa assuming ideal gas behaviour introduces an error of about 3.7%.

The estimated mass of methane lost due to leakage assuming ideal gas behaviour, therefore, introduces an error of about 3.9%. The result obtained assuming ideal gas behaviour is, however, acceptable as a first estimation of the mass of methane lost due to the leakage.

Example 6.12

Extracted values from a Steam Table is given below:

p (bar)	t (°C)	v_g (m^3/kg)	u_f (kJ/kg)	u_g (kJ/kg)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg·K)	s_g (kJ/kg·K)
1.013	100.0	1.673	-	-	419.1	2675.8	1.307	7.355
5	151.8	0.3748	639	2562	640	2749	1.860	6.822
7	165.0	0.2728	696	2573	697	2764	1.992	6.709

Using the above table, determine the following:

(a) Condition of water/steam at 6 bar and $s = 8.3$ kJ/kg·K

(b) Condition of water/steam at 5 bar and $u = 2500$ kJ/kg

(c) Specific internal energy at 1.013 bar and $v = 0.72$ m^3/kg

(d) Specific enthalpy and specific internal energy at 7 bar and 100°C

Solution to Example 6.12

(a) The table above shows that $s = 8.3$ kJ/kg·K is greater than $s_g = 6.822$ kJ/kg·K at 5 bar, and $s_g = 6.709$ kJ/kg·K at 7 bar. From the values of s_g at 5 bar and 7 bar, we can conclude that the saturated vapour specific entropy, s_g, at 6 bar lies in between 6.709 and 6.822, and therefore $s = 8.3$ kJ/kg·K is greater than s_g at 6 bar. That means we have superheated steam at 6 bar and $s = 8.3$ kJ/kg·K.

(b) The table above shows, at 5 bar, $u_f = 639$ kJ/kg and $u_g = 2562$ kJ/kg. Since $u = 2500$ kJ/kg at 5 bar lies between the saturated water specific internal energy u_f and the saturated vapour specific internal energy u_g at 5 bar, we have a mixture of saturated steam and saturated water at 5 bar and $u = 2500$ kJ/kg.

(c) The table above shows, at 1.013 bar, the given specific volume $v = 0.72$ m^3/kg is less than the saturated vapour specific volume $v_g = 1.673$ m^3/kg.

The saturated water specific volume v_f is not given but may be approximated to 1/1000 m^3/kg. The given specific volume $v = 0.72$ m^3/kg lies between v_f and v_g, and therefore what we have is a mixture of saturated steam and saturated water. Using (6.4), we can calculate the dryness fraction x at the given state as follows:

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.72 - 0.001}{1.673 - 0.001} = 0.43$$

The specific internal energy u at the given state could have been determined using (6.5), $u = u_f + x\,(u_g - u_f)$, had we known u_f and u_g. Since these values are not given in the table above, let us evaluate the specific enthalpy h at the given state using (6.6) as follows:

$$h = h_f + x\,(h_g - h_f) = 419.1 + 0.43 \times (2675.8 - 419.1) = 1389.5 \text{ kJ/kg}$$

The specific internal energy at the given state can now be evaluated as follows:

$$u = h - p\,v = 1389.5 \text{ kJ/kg} - 1.013 \text{ bar} \times 0.72 \text{ m}^3\text{/kg} = 1316.5 \text{ kJ/kg}$$

(d) The table above shows, at 7 bar, the saturation temperature is 165.0°C, therefore we have compressed water at 7 bar and 100°C. Since the properties of compressed water is relatively insensitive to the pressure, the compressed water specific enthalpy h at 7 bar and 100°C can be approximated to h_f, the saturated water specific enthalpy, at 100°C. The table above shows that h_f at 100°C is 419.1 kJ/kg, and therefore h at 7 bar and 100°C can be approximated to 419 kJ/kg. Similarly, the compressed water specific internal energy u at 7 bar and 100°C can be approximated to u_f at 100°C, which is in fact not provided in the table above. Let us first calculate u_f at 100°C using $u_f = h_f - p\,v_f$, where $h_f = 419.1$ kJ/kg, $p = 1.013$ bar and v_f may be taken as 1/1000 m^3/kg, which gives $u_f \approx 419$ kJ/kg. Therefore, u at 7 bar and 100°C can be approximated to 419 kJ/kg.

Example 6.13

Specify the condition of water/steam at the following states along with its dryness fraction wherever applicable, and determine the specific internal energy at that state:

(a) At 0.05 bar and 50°C
(b) At 0.75 bar and $s = 7.75$ kJ/kg · K
(c) At 3 bar and 120°C
(d) At 1 bar and $h = 2000$ kJ/kg

Solution to Example 6.13

(a) Saturated Water and Steam Table shows that the saturation temperature at 0.05 bar is 32.9°C. Therefore, 0.05 bar and 50°C is a superheated vapour state. Superheated Steam Table gives $u = 2445$ kJ/kg at that state. Dryness fraction has no meaning at a superheated vapour state.

(b) Saturated Water and Steam Table shows that, at 0.75 bar, $s_f = 1.213$ kJ/kg · K and $s_g = 7.456$ kJ/kg · K. The given value of $s = 7.75$ kJ/kg · K is greater than s_g, which corresponds to saturated steam. Therefore, the given state is a superheated vapour state. Superheated Steam Table shows the entry 0.75 bar and $s = 7.75$ kJ/kg · K at 150°C, and the corresponding entry for u as 2585 kJ/kg. Dryness fraction has no meaning at a superheated vapour state.

(c) Saturated Water and Steam Table shows that the saturation temperature at 3 bar is 133.5°C. Therefore, 3 bar and 120°C is a compressed water state. Using the approximate method provided in the **Solution to Example 6.8**, the compressed water specific internal energy u at 3 bar and 120°C can be approximated to the saturated water specific internal energy u_f at 120°C, which is about 505 kJ/kg as can be read from Saturated Water and Steam Table. Dryness fraction has no meaning at a compressed water state.

(d) Saturated Water and Steam Table shows that, at 1 bar, $h_f = 417$ kJ/kg and $h_g = 2675$ kJ/kg. The given value of $h = 2000$ kJ/kg lies in between h_f and h_g. Therefore, the given state is a mixture of saturated water and saturated steam. The dryness fraction x at that state can be calculated using (6.6) as follows:

$$x = \frac{h - h_f}{h_g - h_f} = \frac{2000 - 417}{2675 - 417} = 0.70$$

The specific internal energy u at the given state is determined using (6.5) as follows:

$$u = u_f + x\,(u_g - u_f) = 417 + 0.70 \times (2506 - 417) = 1879 \text{ kJ/kg}$$

Example 6.14

With the help of a Steam Table, fill in the blank spaces in the following table:

Property	Set 1	Set 2	Set 3	Set 4	Set 5
t, °C	280.8		500		
p, bar		7		80	100
m, kg	1	2	1.3	3.5	3
v, m^3/kg		0.3714			
V, m^3					
Moisture, %				20	
h, kJ/kg	2779				
u, kJ/kg			3077		
s, kJ/kg · K					5.100

Solution to Example 6.14

Set 1 gives the temperature as 280.8°C but does not give the pressure. Each entry on the Steam Table is determined by two properties, and the second property in the case of Set 1 is h = 2779 kJ/kg. When searching for the entry at t = 280.8°C and h = 2779 kJ/kg in a Saturated Water and Steam Table, we will find that it is an entry corresponding to saturated vapour at 65 bar. Other properties for Set 1 could be determined from the entries for saturated vapour at 65 bar and 280.8°C in the Saturated Water and Steam Table.

For Set 2, we can find from any Saturated Water and Steam Table that the specific volume of the saturated steam at 7 bar is 0.2728 m^3/kg. The specific volume that we require is 0.3714 m^3/kg, which is much greater than the specific volume of the saturated steam. This means that the state of Set 2 is a superheated steam state. Any Superheated Steam Table would show that the required entry is at 7 bar and 300°C. Other properties for Set 2 could be determined from the entries at corresponding superheated vapour at 7 bar and 300°C in the Superheated Steam Table.

In case of Set 3, search any Steam Table for an entry that shows the pair 500°C and u = 3077 kJ/kg. It is possible that we find from a Superheated Steam Table that the required entry lies in between the entries corresponding to 60 bar and 500°C, with u = 3081 kJ/kg, and 70 bar and 500°C, with u = 3073 kJ/kg. By appropriate interpolation, we could find the properties required at 500°C and u = 3077 kJ/kg to fill the column for Set 3.

Set 4 has 80 bar and 20% moisture, which is 80% dryness fraction. And, therefore what we have is a mixture of saturated water and saturated steam with $x = 0.8$. The saturation temperature for 80 bar could be found from a Saturated Water and Steam Table as 295.0°C. The properties of saturated water and saturated steam could be read from the Saturated Water and Steam Table at 80 bar, and then (6.4), (6.5), (6.6) and (6.7) can be used to calculate the respective values of v, u, h and s at that state.

For Set 5, the entries corresponding to 100 bar in a Saturated Water and Steam Table would show that the saturated water entropy $s_f = 3.360$ kJ/kg · K and the saturated steam entropy $s_g = 5.615$ kJ/kg · K. The given value of $s =$ 5.100 kJ/kg · K in Set 5 lies between s_f and s_g, and therefore what we have is a mixture of saturated water and saturated steam. Using the values of s, s_f and s_g in (6.7), we can calculate the dryness fraction x. and then (6.4), (6.5) and (6.6) can be used to calculate the respective values of v, u and h. Temperature required is the saturation temperature at 100 bar.

The duly filled table of **Example 6.14** is as follows:

Property	Set 1	Set 2	Set 3	Set 4	Set 5
t, °C	280.8	*300*	500	*295.0*	*311.0*
p, bar	*65*	7	*65*	80	100
m, kg	1	2	1.3	3.5	3
v, m^3/kg	*0.02972*	0.3714	*0.0524*	*0.0191*	*0.0143*
V, m^3	*0.02972*	*0.7428*	*0.0681*	*0.0668*	*0.0428*
Moisture, %	*0*	-	-	20	*22.8*
h, kJ/kg	2779	*3060*	*3415.5*	*2470*	*2425*
u, kJ/kg	*2586*	*2800*	3077	*2317*	*2282*
s, kJ/kg · K	*5.851*	*7.298*	*6.8375*	*5.237*	5.100

Example 6.15

Fill in the blank spaces of the following table with the help of a Steam Table:

Property	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
t, °C	425	200				
p, bar	160	20	70	5		
v, m^3/kg						0.1
x, %					65	
h, kJ/kg			2100			
u, kJ/kg					2008	2940
s, kJ/kg · K				7.4		

Solution to Example 6.15

Property	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
t, °C	425	200	*285.8*	*284.1*	*212.4*	*404*
p, bar	160	20	70	5	*20*	*30*
v, m^3/kg	*0.01573*	*0.001*	*0.0158*	*0.5073*	*0.0651*	0.1
x, %	-	-	*55.35*	-	65	-
h, kJ/kg	*3051*	*852*	2100	*3032*	*2137.5*	*3240*
u, kJ/kg	*2799*	*850*	*1990*	*2779*	2008	2940
s, kJ/kg·K	*5.968*	*2.331*	*4.612*	7.4	*4.978*	*6.934*

Example 6.16

A rigid container of 0.1 m^3 and negligible heat capacity contains steam at 500 kPa and 400^0C. If the container is cooled, determine the pressure and temperature at which steam starts to condense. Determine also the amount of heat removed during the cooling process.

Solution to Example 6.16

The initial state of steam is given by $P_{initial} =$ 500 kPa, and $T_{initial} =$ 400^0C. It is found from a Saturated Water and Steam Table that the saturation temperature at 500 kPa (that is, 5 bar) is 151.8°C. Therefore, we have superheated steam at the initial state, which is cooled until it starts to condense, which means until it reaches the saturated vapour state. We have to determine the pressure and temperature of steam at this final state.

To be able to locate the final state, which is a saturated vapour state, we need to know at least one property at the final state. Since the container is rigid, the total volume remains a constant during the process. Since the given system is a closed one, the total mass remains a constant as well. Therefore, the specific volume remains a constant during the cooling process.

It is found from a Superheated Steam Table that $v_{initial} =$ 0.6172 m^3/kg at 5 bar and 400°C, so that the specific volume of the saturated vapour at the final state is 0.6172 m^3/kg. That is, $v_g =$ 0.6172 m^3/kg. We need to find the corresponding saturation pressure and saturation temperature at this final state. From a Saturated Water and Steam Table we found the following:

$v_g = 0.6253$ m^3/kg at 2.9 bar and 132.4°C.
$v_g = 0.6057$ m^3/kg at 3.0 bar and 133.5°C.

Interpolating between the two states above for v_g of 0.6172 m^3/kg, we get the saturation values as follows:

$$\frac{0.6172 - 0.6057}{0.6253 - 0.6057} = \frac{P_{sat} - 3.0}{2.9 - 3.0} = \frac{T_{sat} - 133.5}{132.4 - 133.5}$$

which gives $T_{sat} = 132.9$°C, and $P_{sat} = 2.94$ bar. The superheated steam at 5 bar and 400°C, when cooled, starts to condense at 2.94 bar and 132.9°C.

To determine the amount of heat removed from the system, we shall use the first law. Since the container is rigid and since no other form of work is done on the system, the first law applied to the given system yields

$$Q_{in} = \Delta U = m_{steam}\left(u_{final} - u_{initial}\right)$$

Superheated Steam Table gives $u_{initial} = 2963$ kJ/kg at 5 bar and 400°C. We need to carry out an interpolation to determine u_{final}, which is u_g at 2.94 bar and 132.9°C.

Referring to a Saturated Steam and Water Table for u_g values, we got the following:
$u_g = 2543$ kJ/kg and $v_g = 0.6253$ m^3/kg at 2.9 bar and 132.4°C.
$u_g = 2544$ kJ/kg and $v_g = 0.6057$ m^3/kg at 3.0 bar and 133.5°C.

Interpolating between the two states yields the following:

$$\frac{0.6172 - 0.6057}{0.6253 - 0.6057} = \frac{u_{final} - 2544}{2543 - 2544}$$

which gives $u_{final} = 2543.4$ kJ/kg.

The mass of the steam can be found as follows:

$$m_{steam} = \frac{\text{total volume}}{\text{specific volume}} = \frac{0.1 \text{ m}^3}{0.6172 \text{ m}^3/\text{kg}} = 0.1620 \text{ kg}$$

Therefore,

$$Q_{in} = (0.1620 \text{ kg}) \times (2543.4 - 2963) \ \text{kJ/kg} = -67.975 \text{ kJ}$$

That is, the heat removed during the cooling process is about 68 kJ.

Example 6.17

A rigid tank with negligible heat capacity, is perfectly insulated and divided into two unequal compartments A and B separated by a partition. Compartment A, having 0.05 m^3 volume, contains steam at 1.5 bar and 250°C. Compartment B, having 0.15 m^3 volume, contains steam at 8 bar and 250°C. The partition is removed and the system is allowed to reach equilibrium. Determine the pressure and the temperature at the new equilibrium state.

Solution to Example 6.17

Application of the first law of thermodynamics to the entire system would give

$$\Delta U = 0$$

since no heat or work transfers occur between the entire system and the surroundings. Neglecting the internal energy changes of the tank and the partition, we shall write the above as

$$U_{final} = U_{initial} \tag{6.10}$$

for the steam present in the tank.

We shall determine $U_{initial}$ as follows:

$$\begin{aligned} U_{initial} &= m_A u_A + m_B u_B \\ &= \frac{0.05 \text{ m}^3}{v_A} u_A + \frac{0.15 \text{ m}^3}{v_B} u_B \end{aligned} \tag{6.11}$$

of which v_A and u_A are the respective specific volume and specific internal energy at 1.5 bar and 250°C, and v_B and u_B are the respective specific volume and specific internal energy at 8 bar and 250°C.

Saturated Water and Steam Table would show that the saturation temperature is 111.4°C at 1.5 bar and 170.4°C at 8 bar. It is therefore we conclude that the steam in each of the two compartments are at superheated state. Using the Superheated Steam Table, we can get the following:

$v_A = 1.601$ m^3/kg and $u_A = 2733$ kJ/kg at 1.5 bar and 250°C.
$v_B = 0.2933$ m^3/kg and $u_A = 2716$ kJ/kg at 8 bar and 250°C.

Using these values in (6.11), we get $U_{initial} = 1474$ kJ. Substituting it in (6.10), we get $U_{final} = 1474$ kJ.

The specific internal energy at the final state is therefore

$$\begin{aligned} u_{final} &= \frac{U_{final}}{m_A + m_B} \\ &= \frac{1474 \text{ kJ}}{[(0.05/1.601) + (0.15/0.2933)] \text{ kg}} = 2716 \text{ kJ/kg} \end{aligned}$$

We need one more property at the final state to completely specify the final state. The other property is in fact the specific volume at the final state determined as follows:

$$\begin{aligned} v_{final} &= \frac{V_{total}}{m_A + m_B} \\ &= \frac{[0.05 + 0.15] \text{ m}^3}{[(0.05/1.601) + (0.15/0.2933)] \text{ kg}} = 0.3686 \text{ m}^3\text{/kg}. \end{aligned}$$

Searching for an entry with $u = 2716$ kJ/kg and $v = 0.3686$ m^3/kg all over the Steam Table, we come across these adjacent entries for the superheated steam at 6 bar.

$v = 0.3522$ m^3/kg and $u = 2640$ kJ/kg at 6 bar and 200°C.
$v = 0.3940$ m^3/kg and $u = 2722$ kJ/kg at 6 bar and 250°C.

Interpolating for u at $v = 0.3686$ m^3/kg gives

$$\begin{aligned} \frac{u - 2640}{2722 - 2640} &= \frac{0.3686 - 0.3522}{0.3940 - 0.3522} \\ u &= 2672 \text{ kJ/kg} \end{aligned}$$

which is less than 2716 kJ/kg. Thus, this entry does not give the state required.

Let us now try the adjacent entries for the superheated steam at 7 bar.

$v = 0.3001$ m^3/kg and $u = 2636$ kJ/kg at 7 bar and 200°C.
$v = 0.3364$ m^3/kg and $u = 2720$ kJ/kg at 7 bar and 250°C.

Interpolating for u at $v = 0.3686$ m^3/kg gives the following:

$$\begin{aligned} \frac{u - 2636}{2720 - 2636} &= \frac{0.3686 - 0.3001}{0.3364 - 0.3001} \\ u &= 2795 \text{ kJ/kg} \end{aligned}$$

which is more than 2716 kJ/kg. Thus, this entry does not give the state required.

Searching between 6 bar and 7 bar in the range of 200°C to 250°C by interpolation, we get the required v-u combination at 6.4 bar and 247°C.

6.7 Summary

In working out thermodynamic problems on closed systems with water/steam, the following tools are used:

Tool 1: Principle of conservation of mass.

Tool 2: The first law of thermodynamics applied to closed simple compressible systems, given by $Q_{in}+W_{in}=\Delta U$, which is the principle of conservation of energy.

Tool 3: When two properties at an equilibrium state are known, the remaining properties at that state may be obtained from the Steam Table.

Tool 4: Evaluation of ΔU using data on internal energy at the initial and final states obtained from the Steam Table.

Tool 5: Any other constraint stated in the problem.