Chapter 5

WORKING WITH IDEAL GAS

> In thinking about the world we are fixed with the same kind of problems as a cartographer who tries to cover the curved face of the earth with a sequence of plane maps. We can only expect an approximate representation of reality from such a procedure and all rational knowledge is therefore necessarily limited.
>
> – Fritjof Capra (*The Tao of Physics*)

In this chapter, we will learn to apply the first law of thermodynamics to closed, simple compressible systems containing ideal gas, which is indeed an imaginary gas. However, the behaviour of real gases at low pressures are often approximated to the behaviour of an ideal gas, the properties of which are related to each other in a very special way. We will also learn about these relationships in this chapter.

5.1 Definition of an Ideal Gas

The volume occupied by the molecules of an ideal gas is assumed to be negligibly small compared to the total gas volume. Also, molecules of an ideal gas are assumed to have no influence on each other. Hence, an ideal gas satisfies the following two conditions:

- An ideal gas at an equilibrium state can be described by the ideal gas equation of state,

  $$PV = nRT \tag{5.1}$$

  where P is the **absolute pressure**, V is the volume, n is the amount of gas present, R is the **universal gas constant**, and T is the **absolute temperature**. Absolute pressure is the actual pressure at a point. When we measure the pressure at a point using most pressure-measuring devices which read zero in the local atmosphere, what we get is the **gauge pressure**. Absolute pressure is then obtained by adding the local atmospheric pressure to the gauge pressure. For a description on absolute temperature, see Section 5.2. For information on the universal gas constant R, see Section 5.3.

- The specific internal energy of an ideal gas is a function of temperature only. That is to say

  $$u = u(T) \quad \text{for an ideal gas.} \tag{5.2}$$

  In other words, the internal energy of an ideal gas experiences absolutely no change when the volume occupied by an ideal gas or the pressure exerted on an ideal gas is varied so as to change the intermolecular distances, while maintaining the temperature constant. It should not be surprising, because we know that intermolecular phenomena are totally absent in an ideal gas, and thus altering the distances among the molecules of an ideal gas shall have absolutely no influence on the internal energy content of an ideal gas. However, changing the temperature of an ideal gas alters the behaviour of the individual gas molecules, and therefore the internal energy is changed.

The behaviour of real gases at low pressures can be approximated by (5.1) and (5.2). It is therefore, a real gas at low pressures may be considered an ideal gas.

5.2 Absolute Temperature Scale

The scales of temperature in the SI units are the Celsius scale (°C), which is also known as centigrade scale, and the Kelvin scale (K). Temperatures in these scales are related to each other by

$$T_K \ = T_C + 273.15$$

where T_K is the temperature in the Kelvin scale and T_C is its equivalent in the Celsius scale.

Kelvin scale is an **absolute temperature scale**, which implies the lowest attainable temperature in the scale is zero. There are no negative temperatures on an absolute temperature scale. The concept of absolute temperature is one of the most confusing point in the subject of thermodynamics for a beginner of this subject. The concept of absolute temperature will become clear when one goes on to learn the second law of thermodynamics. For now, let us accept the concept of an absolute temperature scale without further elaboration and proceed.

5.3 Different Forms of the Ideal Gas Equation of State

- When using the ideal gas equation of state in the form given by (5.1), we take P in kPa, V in m^3, and T in K. The amount of gas present, denoted by n, is taken in kmol. And, R takes the value 8.314 kJ/kmol · K for any gas, and therefore it is known as the **universal gas constant**.
- The ideal gas equation of state is also used in the form

  $$PV \ = \ mRT \tag{5.3}$$

  where P is in kPa, V is in m^3, and T is in K. The mass of gas present, denoted by m, is taken in kg. Therefore R takes the unit kJ/kg · K. When R is in kJ/kg · K, it is known as the **specific gas constant**.

- We could also write the ideal gas equation of state in the form

  $$Pv = RT \tag{5.4}$$

  where P is in kPa, and T is in K. If $v = V/n$ is the molar volume in m^3/kmol, then R is the universal gas constant, taking the value 8.314 kJ/kmol · K for all gases. If $v = V/m$ is the specific volume in m^3/kg, then R is the specific gas constant in kJ/kg · K.

Combining (5.1) and (5.3), we relate the universal gas constant and the specific gas constant by

$$n \times \text{ universal gas constant} = m \times \text{ specific gas constant}$$

which gives

$$\text{specific gas constant} = \frac{\text{universal gas constant}}{M}$$

where $M = m/n$ is known as the **molar mass**, and it takes the unit kg/kmol. The universal gas constant takes the value 8.314 kJ/kmol · K for all gases, and the value of molar mass is specific to the gas. Therefore, the numerical value of the specific gas constant, which depends on the molar mass of the gas, is specific to the gas and its value changes from one gas to the other. Please note it is common to use the same symbol R for both the universal gas constant and the specific gas constant, since the units will determine which one is in use.

Keep in mind that the numerical value of molar mass is the same as that of the relative molecular mass, whereas molar mass takes the unit g/mol or kg/kmol and the relative molecular mass is dimensionless.

5.4 Internal Energy and C_v

The change in the specific internal energy of **any substance** with respect to temperature while the volume is maintained constant can be expressed as

$$\begin{aligned} C_v &= \lim_{\delta T \to 0} \frac{u(v, T+\delta T) - u(v,T)}{\delta T} \\ &= \lim_{\delta T \to 0} \left. \frac{u(T+\delta T) - u(T)}{\delta T} \right|_{\text{at constant volume}} \\ &= \left. \frac{du}{dT} \right|_v \end{aligned} \qquad (5.5)$$

which is the definition of C_v known as the **specific heat at constant volume**. It is one of the most extensively used specific heats in thermodynamics.

Since u of an ideal gas is a function of T only, u is independent of its volume. Therefore, the definition of C_v given by (5.5) simplifies to

$$C_v = \lim_{\delta T \to 0} \frac{u(T+\delta T) - u(T)}{\delta T} = \frac{du}{dT} \quad \text{for an ideal gas.} \qquad (5.6)$$

A visual representation of the specific heat at constant volume is given in Figure 5.1, which shows a typical plot of u versus T for an ideal gas. We know that the slope of the tangent to the u versus T curve at point A, shown by the dashed line in Figure 5.1, represents an infinitesimal change in u with T at point A. This slope quantifies the specific heat at constant volume at the temperature represented by point A. Since the slope may vary from one temperature to another, it is obvious that C_v may also vary with temperature. Since u is a function of T alone for an ideal gas, C_v of an ideal gas is a function only of T.

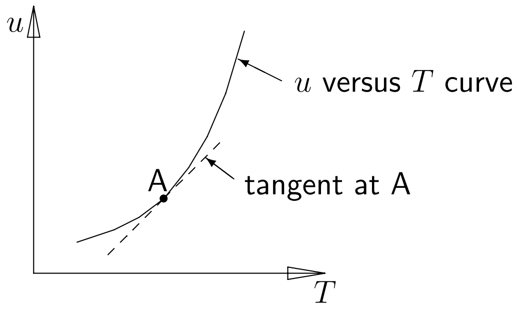

Figure 5.1 An example of u versus T curve.

The unit for C_v used in this textbook is kJ/kg · K if u in (5.6) is taken as the specific internal energy. The unit for C_v used is kJ/kmol · K if u in (5.6) is taken as the molar internal energy. Since a unit change in the Kelvin scale (K) is the same as a unit change in the Celsius scale (°C), the numerical value of C_v in kJ/kg · K is identical to that in kJ/kg · °C. Similarly, the numerical value of C_v in kJ/kmol · K is the same as that in kJ/kmol · °C.

Equation (5.6) for an ideal gas can be rearranged to give

$$du = C_v \, dT \tag{5.7}$$

which is also applicable for any substance undergoing a constant-volume process, see (5.5). Integrating (5.7) between the initial and final equilibrium states of a process, we get

$$\int_{u_o}^{u_f} du = \int_{T_o}^{T_f} C_v \, dT$$

where u_o and u_f are the respective specific (or molar) internal energies at the initial and the final equilibrium states of the process, and T_o and T_f are the respective temperatures at the initial and the final equilibrium states of the process.

Since u is a property, the above reduces to

$$\Delta u \; = \; u_f - u_o \; = \; \int_{T_o}^{T_f} C_v \, dT \tag{5.8}$$

which can be used to evaluate Δu for an ideal gas undergoing any process, constant-volume or not, or for any substance undergoing a constant-volume process.

If C_v of (5.8) is in kJ/kg · K, then

$$\Delta U \; = \; m \, \Delta u \; = \; m \int_{T_o}^{T_f} C_v \, dT$$

where m is the mass of the substance in kg. If C_v of (5.8) is in kJ/kmol · K, then

$$\Delta U \; = \; n \, \Delta u \; = \; n \int_{T_o}^{T_f} C_v \, dT$$

where n is the amount of the substance in kmol.

5.5 Enthalpy and C_p

The change in the specific enthalpy of **any substance** with respect to temperature while the pressure is maintained constant is expressed as

$$\begin{aligned} C_p &= \lim_{\delta T \to 0} \frac{h(P, T+\delta T) - h(P,T)}{\delta T} \\ &= \lim_{\delta T \to 0} \left. \frac{h(T+\delta T) - h(T)}{\delta T} \right|_{\text{at constant pressure}} \\ &= \left. \frac{dh}{dT} \right|_P \end{aligned} \tag{5.9}$$

which is the definition of C_p known as the **specific heat at constant pressure**. It is another extensively used specific heat in thermodynamics.

The specific enthalpy h given by (4.2) becomes

$$h = u + RT \qquad \text{for an ideal gas,} \tag{5.10}$$

when the Pv term of (4.2) is eliminated using (5.4). Since u at an equilibrium state of an ideal gas is a function of temperature alone and R is a constant, it is obvious from (5.10) that h of an ideal gas is also a function of temperature alone. That is to say

$$h = h(T) \quad \text{for an ideal gas.}$$

Since h of an ideal gas is a function of T only, it is independent of its pressure. Therefore, C_p of (5.9) simplifies to

$$C_p = \lim_{\delta T \to 0} \frac{h(T+\delta T) - h(T)}{\delta T} = \frac{dh}{dT} \qquad \text{for an ideal gas.} \tag{5.11}$$

Like C_v, C_p may also vary with temperature. The variation of C_p with temperature would, of course, be different for different substances. Since h is a function of T alone for an ideal gas, C_p of an ideal gas is a function of T only. The units of C_p is the same as that of C_v (discussed in Section 5.4.)

Equation (5.11) for an ideal gas can be rearranged to give

$$dh = C_p \, dT \tag{5.12}$$

which is also applicable for any substance undergoing a constant-pressure process, see (5.9). Integrating (5.12) between the initial and final equilibrium states of a process, we get

$$\int_{h_o}^{h_f} dh = \int_{T_o}^{T_f} C_p \, dT$$

where h_o and h_f are the respective specific (or molar) enthalpies at the initial and the final equilibrium states of the process, and T_o and T_f are the respective temperatures at the initial and the final equilibrium states of the process.

Since h is a property, the above reduces to

$$\Delta h \;=\; h_f - h_o \;=\; \int_{T_o}^{T_f} C_p \, dT \tag{5.13}$$

which can be used to evaluate Δh for an ideal gas undergoing any process, constant-pressure or not, or for any substance undergoing a constant-pressure process.

If C_p of (5.13) is in kJ/kg · K, then

$$\Delta H \;=\; m \, \Delta h \;=\; m \int_{T_o}^{T_f} C_p \, dT$$

where m is the mass of the substance in kg. If C_p of (5.13) is in kJ/kmol · K, then

$$\Delta H \;=\; n \, \Delta h \;=\; n \int_{T_o}^{T_f} C_p \, dT$$

where n is the amount of the substance in kmol.

5.6 Relating Ideal Gas Specific Heats

The specific heats of an ideal gas are related to each other in a very simple manner. Let us now see how to obtain this relationship. Take the expression for the specific enthalpy of an ideal gas, $h = u + RT$, given by

(5.10), and differentiate it with respect to temperature T. Since both h and u are functions of temperature alone for an ideal gas, we get

$$\frac{dh}{dT} = \frac{du}{dT} + R \qquad \text{for an ideal gas.}$$

Substituting $C_v = du/dT$ of (5.6) and $C_p = dh/dT$ of (5.11) in the above expression, we get

$$C_p = C_v + R \qquad \text{for an ideal gas.} \tag{5.14}$$

If the specific heats are in kJ/kmol · K then R is the universal gas constant. If the specific heats are in kJ/kg · K then R is the specific gas constant.

The ratio between the specific heats is known as the **specific heat ratio** γ, and is defined as

$$\gamma \equiv \frac{C_p}{C_v} \tag{5.15}$$

The specific heat ratio is also known as the **isentropic exponent**, and is sometimes denoted by k.

Combining (5.14) and (5.15) so as to eliminate C_p, we get

$$C_v = \frac{R}{\gamma - 1} \qquad \text{for an ideal gas.} \tag{5.16}$$

Combining (5.15) and (5.16) so as to eliminate C_v, we get

$$C_p = \frac{\gamma R}{\gamma - 1} \qquad \text{for an ideal gas.} \tag{5.17}$$

5.7 Data on Ideal Gas Specific Heats

Ideal gas specific heats are sensitive to changes in temperature. However, the effect of temperature is negligibly small on the specific heats of monoatomic gases. Therefore, for monoatomic gases, such as argon, helium, etc., C_v = 12.5 kJ/kmol · K and C_p = 20.8 kJ/kmol · K could be used independent of temperature. Specific heats change rather slowly with temperature for diatomic gases, such as O_2, H_2, and N_2. For polyatomic gases, such as CO_2, CH_4, C_2H_6, etc, specific heats vary significantly with temperature.

Data obtained for C_p of a gas at different temperatures while maintaining low pressures, can be fitted using algebraic equations such as the one given below as a function of temperature:

$$C_p = a + bT + cT^2 + dT^3$$

where the constants a, b, c and d take different values for different gases as shown in Table 5.1. Algebraic expressions for C_p are available in the literature* for a large number of gases that are assumed to behave as ideal gas. Substituting the algebraic equation describing C_p in (5.14), we can get the algebraic equation for C_v as a function of T.

Gas	a	$b \times 10^2$	$c \times 10^5$	$d \times 10^9$
Hydrogen	29.11	-0.1916	0.4003	-0.8704
Oxygen	25.48	1.520	-0.7155	1.312
Air	28.11	0.1967	0.4802	- 1.966
Carbon dioxide	22.26	5.981	-3.501	7.469
Water vapour	32.24	0.1923	1.055	-3.595

Table 5.1 Values of the constants in $C_p = a + bT + cT^2 + dT^3$ for a few selected gases valid in the temperature range 273 K to 1800 K, where C_p is in kJ/kmol · K and T is in K.

Table 5.2 shows the values of ideal gas specific heats of few selected gases at 300 K. Specific heats are given both in kJ/kg · K and in kJ/kmol · K. Multiplying the specific heat given in kJ/kg · K by the molar mass, we get the specific heat given in kJ/kmol · K. The last column of Table 5.2 shows the values of the specific heat ratio γ.

This table also shows that the specific heats of monoatomic gases are almost the same, when taken in the unit of kJ/kmol · K, and the specific heats of diatomic gases are also nearly the same. It is common to assume that air also falls into the group of diatomic gases. For polyatomic gases,

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*See, for example, Table A-2 of Çengel, Y.A. & Boles, M.A. 1998 Thermodynamics: an engineering approach, 3^{rd} Edition, McGraw-Hill International Editions.

specific heats differ from gas to gas. Note that specific heats of ideal gases are also known as **zero-pressure specific heats**, and are sometimes denoted by C_{vo} and C_{po}, since all real gases approach ideal-gas behaviour at low (or near zero) pressures.

GAS molar mass (in kg/kgmol)	C_p	C_v	C_p	C_v	γ
	(in kJ/kg · K)		(in kJ/kmol · K)		
Argon 39.950	0.5203	0.3122	20.786	12.472	1.666
Helium 4.003	5.1930	3.1159	20.788	12.473	1.666
Hydrogen 2.016	14.3230	10.1987	28.875	20.561	1.404
Nitrogen 28.013	1.0400	0.7432	29.134	20.819	1.399
Air 28.97	1.005	0.718	29.105	20.793	1.400
Carbon monoxide 28.010	1.0410	0.7442	29.158	20.845	1.399
Carbon dioxide 44.010	0.8457	0.6568	37.219	28.906	1.288
Ethane 30.070	1.7668	1.4903	53.128	44.813	1.186

Table 5.2 Ideal gas specific heats at 300 K for a few selected gases.

It should be borne in mind that for real gases, the specific heats depend not only on the temperature but also on the pressure or volume, and for incompressible substances, such as liquids and solids, both constant-pressure and constant-volume specific heats are approximately the same, and thus $C_p \approx C_v$.

5.8 Evaluation of ΔU for an Ideal Gas

In this section we will learn to evaluate ΔU, the internal energy difference, of an ideal gas in three different ways.

5.8.1 Using an Expression for C_v

Suppose you are asked to evaluate the specific internal energy change of nitrogen when its temperature is increased from 300 K to 600 K at low pressures given the expression

$$C_p/R = a + bT + cT^2 + dT^3 + eT^4$$

where $a = 3.675$, $b = -1.208\times10^{-3}$, $c = 2.324\times10^{-6}$, $d = -0.632\times10^{-9}$, $e = -0.226\times10^{-12}$, C_p takes the unit of R, and T is in K.

Since nitrogen is held at low pressure, it is assumed to behave as an ideal gas. Substituting the algebraic equation describing C_p in (5.14), we get

$$\begin{aligned} C_v &= (a + b\,T + c\,T^2 + d\,T^3 + e\,T^4)\,R - R \\ &= (a-1)\,R + b\,RT + c\,RT^2 + d\,RT^3 + e\,RT^4 \end{aligned}$$

Substituting it in (5.8) and integrating the resulting expression from 300 K to 600 K, we get

$$\begin{aligned} \Delta u &= (a-1)\,R\,(600-300) \\ &\quad + b\,R\left(\frac{600^2-300^2}{2}\right) + c\,R\left(\frac{600^3-300^3}{3}\right) \\ &\quad + d\,R\left(\frac{600^4-300^4}{4}\right) + e\,R\left(\frac{600^5-300^5}{5}\right) \end{aligned}$$

Using the given numerical values of a, b, c, d and e, and $R = 8.314$ kJ/kmol·K in the above, we get $\Delta u = 6345.5$ kJ/kmol.

The accuracy of the result obtained depends, of course, on the accuracy of the algebraic equation used. This method is, however, a very inconvenient method for hand calculations.

5.8.2 Using the Ideal-Gas Property Table

Alternatively, we can use the **ideal-gas property tables**, similar to Table 5.3, to evaluate Δu of nitrogen as

$$\begin{aligned}\Delta u &= u_{\text{at 600 K}} - u_{\text{at 300 K}}\\ &= 12574\ \text{kJ/kmol} - 6229\ \text{kJ/kmol} = 6345\ \text{kJ/kmol}\end{aligned}$$

In the above calculations, Δu is evaluated using the u values obtained from ideal-gas property tables that are compiled by integrating (5.7) with the help of computers using very accurate algebraic equations describing C_v as a function of temperature. This method therefore gives accurate values of Δu. Also, it is very easy to use.

T	h	u
0	0	0
220	6,391	4,562
230	6,683	4,770
240	6,975	4,979
250	7,266	5,188
300	8,723	6,229
-	-	-
-	-	-
600	17,563	12,574
-	-	-
-	-	-

Table 5.3 Ideal gas enthalpy and internal energy for N_2, where T is in K, and h and u are in kJ/kmol$\cdot$K.

An important aspect of the ideal gas property tables, such as Table 5.3 and others[†], is that the entries do not include pressure as an entry. It is

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[†] See, for example, the following tables:

Table A-17 to Table A-25 of Çengel, Y.A. & Boles, M.A. 1998 Thermodynamics: an engineering approach, 3rd Edition, McGraw-Hill International Editions

Table A-5M to Table A-11M of Wark, K. 1989 Thermodynamics, 5th Edition, McGraw-Hill International Editions.

because data on ideal gas properties of gases are meaningful only at low pressures since ideal gas behaviour is imitated by real gases only at low pressures.

Another aspect of the individual entries of internal energy and enthalpy in the ideal gas property tables is that individual entries of u and h depend on the respective reference states chosen as described below. Let us set the reference temperature as T_{ref}, and the corresponding u and h values are, say, u_{ref} and h_{ref}, respectively. Integrating (5.7) and (5.12) between T_{ref} and T, we get

$$u = u_{ref} + \int_{T_{ref}}^{T} C_v \, dT$$

$$h = h_{ref} + \int_{T_{ref}}^{T} C_p \, dT$$

The reference state chosen may differ from one thermodynamic property table to the other. For example, the reference state chosen in Table 5.3 is 0 K, and the h and u values at this reference state are set to zero. The individual entries of u and h at a chosen T may differ from one table to another, since the individual entry depends on the reference state. However, the change in internal energy or enthalpy calculated using individual entries from a particular table is independent of the reference state, as shown below for the case of Δu as

$$\begin{aligned} \Delta u &= u_2 - u_1 \\ &= \left(u_{ref} + \int_{T_{ref}}^{T_2} C_v \, dT \right) - \left(u_{ref} + \int_{T_{ref}}^{T_1} C_v \, dT \right) \\ &= \int_{T_1}^{T_2} C_v \, dT \end{aligned}$$

5.8.3 Using an Average Value for C_v

Yet another, but an approximate method for the evaluation of Δu is to take C_v as a constant about an average value, $(C_v)_{av}$, over the temperature range of our interest, which reduces (5.8) to the following approximate relationship:

$$\Delta u \approx (C_v)_{av}\,(T_f - T_o) \tag{5.18}$$

Similarly, if C_p is assumed to be a constant about an average value, $(C_p)_{av}$, over the temperature range of our interest, then (5.13) reduces to the following approximate relationship:

$$\Delta h \approx (C_p)_{av}\,(T_f - T_o) \tag{5.19}$$

Suppose $(C_v)_{\text{at 300 K}}$ and $(C_v)_{\text{at 600 K}}$ are given as 0.743 kJ/kg · K and 0.778 kJ/kg · K, respectively. Since these values are somewhat close, we can evaluate the algebraic mean of the C_v values given at 300 K and 600 K, which gives $(C_v)_{\text{av}} = 0.761$ kJ/kg · K. Using this value in (5.18), we get

$$\begin{aligned}\Delta u &= 0.761 \times (600 - 300) \text{ kJ/kg}\\ &= (228.3 \text{ kJ/kg}) \times (28.013 \text{ kg/kmol})\\ &= 6395 \text{ kJ/kmol}\end{aligned}$$

Since Δu is evaluated using the average value for C_v, the result is only an approximate result. Yet, this method is widely used owing to its simplicity, particularly when hand calculations are carried out. However, the above method may give inaccurate results if the temperature interval considered is very large.

5.9 Worked Examples

The following tools are useful in working out thermodynamic problems on closed simple compressible systems containing ideal gas:

Tool 1: The first law of thermodynamics applied to closed simple compressible systems, given by $Q_{in} + W_{in} = \Delta U$, which is also the principle of conservation of energy.

Tool 2: Evaluation of ΔU using data on specific heats, or data on internal energy itself.

Tool 3: The ideal gas equation of state that interrelates pressure, temperature, volume and mass (or amount of substance) of an ideal gas.

Example 5.1

A closed rigid container, shown in Figure 5.2, with negligible heat capacity has a volume of 1 m^3. It holds air at 300 kPa and 303 K. Heat is supplied until the temperature of air reaches 500 K. Take C_v for air as 0.718 kJ/kg·K. The molar mass of air is 29 kg/kmol. Determine the amount of heat supplied to air assuming air behaves as an ideal gas.

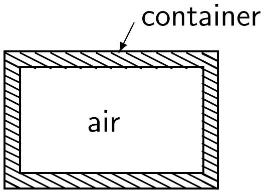


Figure 5.2

Solution to Example 5.1

The amount of heat supplied to the air, denoted by Q_{in}, should be found. Air in the closed container is taken as a closed system. The first law applied to the closed system, that is air, gives $Q_{in} + W_{in} = \Delta U$. The container is rigid and therefore the shape and size of its boundary cannot be changed. Thus, no work is done at the boundary of the system in changing its shape or size and thereby compressing or expanding the air contained in the container. There is also no other forms of work involved, and therefore $W_{in} = 0$. Thus, the first law gives

$$Q_{in} = \Delta U \tag{5.20}$$

If we can evaluate ΔU by some means, then Q_{in} will be known from (5.20). Since air is assumed to behave as an ideal gas, (5.8) is used to evaluate Δu as

$$\begin{aligned} \Delta u &= \int_{303\ K}^{500\ K} (0.718 \text{ kJ/kg}\cdot\text{K})\, dT \\ &= (0.718 \text{ kJ/kg}\cdot\text{K}) \times (500 \text{ K} - 303 \text{ K}) = 141.5 \text{ kJ/kg} \end{aligned}$$

Therefore,

$$\Delta U = 141.5 \times m \text{ kJ} \tag{5.21}$$

where m is the mass of air in the closed tank in kg, and the value of m is not known.

There is only one way to calculate m, which is to use the ideal gas equation of state in the form $PV = mRT$, in which R is the specific gas constant calculated as

$$R = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{29 \text{ kg/kmol}} = 0.287 \text{ kJ/kg}\cdot\text{K}$$

We know the values of P, V and T at the initial state of the system, and therefore we work out the value of m as

$$m = \left(\frac{P\,V}{R\,T}\right) = \frac{(300 \text{ kPa})\ (1 \text{ m}^3)}{(0.287 \text{ kJ/kg}\cdot\text{K})\ (303 \text{ K})} = 3.45 \text{ kg} \tag{5.22}$$

Substituting m of (5.22) in (5.21), we get

$$\Delta U = 141.5 \times 3.45 = 488 \text{ kJ} \tag{5.23}$$

Combining (5.20) and (5.23) so as to eliminate ΔU, we get $Q_{in} = 488$ kJ which is the amount of heat supplied to the air to increase its temperature from 303 K to 500 K.

Example 5.2

If the closed rigid container of Example 5.1 weighs 10 kg and is made of steel having the specific heat 0.5 kJ/kg · °C, determine the amount of heat supplied to increase the temperature of air to 500 K.

Solution to Example 5.2

The closed rigid container of Example 5.1 is made of a material with negligible heat capacity, and therefore the container absorbs a negligible amount of heat as we heat the air to reach 500 K. Thus, we ignored the amount of heat supplied to the container, and evaluated only the amount of heat supplied to the air to increase its temperature to 500 K.

In this example, the container is said to be made of steel. So, as we heat the air to 500 K, the container also is heated to 500 K absorbing part of the heat supplied. We, therefore, have to take the closed system to include both the air and the container. Let us denote the amount of heat supplied to the closed system as Q_{in}, and we need to evaluate the value of Q_{in}.

Since there are no work modes involved in the process, $W_{in} = 0$. Thus, the first law applied to the closed system gives

$$Q_{in} = \Delta U \tag{5.24}$$

where ΔU is made up of the following components:

$$\Delta U = \Delta U_{air} + \Delta U_{container} \tag{5.25}$$

Since air in this example undergoes the same property changes as the air in Example 5.1, ΔU_{air} is evaluated following the same steps as in Example 5.1. Therefore, we get

$$\Delta U_{air} = 488 \text{ kJ}$$

$\Delta U_{container}$ is evaluated as

$$\begin{aligned}\Delta U_{container} &= \text{mass} \times \text{specific heat} \times \text{temperature increase}\\ &= (10 \text{ kg}) \times (0.5 \text{ kJ/kg}\cdot{}^\circ\text{C}) \times (500 \text{ K} - 303 \text{ K})\\ &= 985 \text{ kJ}\end{aligned}$$

Substituting the above in (5.25), we get

$$\Delta U = 488 \text{ kJ} + 985 \text{ kJ} = 1473 \text{ kJ}$$

which is used in (5.24) to obtain

$$Q_{in} = 1473 \text{ kJ}$$

which is the amount of heat supplied to increase the temperature of the air from 303 K to 500 K when the air is contained in a steel container of 10 kg mass and 0.5 kJ/kg · °C specific heat.

Let us closely examine the equation

$$\Delta U_{container} = \text{mass} \times \text{specific heat} \times \text{temperature increase}$$

which was used to determine the internal energy increase of the container in **Example 5.2**. The term specific heat used here can be either C_v or C_p, since $C_p \approx C_v$ for incompressible substances, such as liquids and solids. Since specific heats are sometimes referred to as the heat capacities, the statement "container with negligible heat capacity" in **Example 5.1** simply means the specific heat of the material of construction of the container is negligibly small. So $\Delta U_{container}$ is negligibly small for a container with negligible heat capacity.

Example 5.3

Helium gas is contained in a rigid vessel of 0.5 m^3 at 500 kPa. The gas is agitated violently by a stirrer that transfers 250 kJ of work to the gas. Assuming that the system undergoes an adiabatic process, determine the final pressure of the gas. Assume that C_v for helium is a constant at 12.46 kJ/kmol·K.

Solution to Example 5.3

The final pressure of helium, denoted by P_f, could be calculated only in one way, that is, by using the ideal gas equation of state at the final state as $P_f V_f = n R T_f$. Since the vessel is rigid, the volume of the vessel remains a constant. Therefore, $V_f = 0.5$ m^3. Substituting the known values in the above equation, we get

$$P_f \times 0.5 = 8.314 \, n \, T_f \tag{5.26}$$

where P_f is in kPa, n is in kmol and T_f is in K. Calculation of P_f from (5.26) is possible only if we know the values of n and T_f.

Let us calculate the value of n using the ideal gas equation of state at the initial state as $P_o V_o = n R T_o$, in which $P_o = 500$ kPa, $V_o = 0.5$ m^3, and T_o is not known. Therefore, we get

$$500 \times 0.5 = 8.314 \, n \, T_o \tag{5.27}$$

where n is in kmol as told already, and T_o is in K.

There are two independent equations (5.26) and (5.27), and there are four unknowns which are P_f, T_o, T_f and n. We require two more independent equations to solve for the unknowns.

We have been told at the beginning of this section that we can use three tools. And, we have so far used only one which is the ideal gas equation of state. Let us examine the problem statement to see if the first law of thermodynamics can be used in this example.

It is said that the system undergoes an adiabatic process, and therefore $Q_{in} = 0$. It is also said that the system has rigid boundary and that 250 kJ of stirring work is done on the system. Therefore, $W_{in} = 250$ kJ. The first law applied to closed system, $Q_{in} + W_{in} = \Delta U$, thus gives

$$\Delta U = 250 \text{ kJ} \tag{5.28}$$

Now, there are three independent equations (5.26), (5.27) and (5.28), and five unknowns P_f, T_o, T_f, n and ΔU. We still require two more independent equations to solve for the unknowns.

Out of the three tools, we have so far used two which are the ideal gas equation of state and the first law of thermodynamics. The remaining tool is the evaluation of ΔU using data on specific heats, or data on internal energy itself.

Since the value of C_v is given in the problem statement and since helium behaves as an ideal gas, we evaluate ΔU using (5.8) as

$$\begin{aligned} \Delta U &= (n \text{ kmol}) \int_{T_o\ K}^{T_f\ K} (12.46 \text{ kJ/kmol}\cdot\text{K})\, dT \\ &= 12.46\, n\, (T_f - T_o) \text{ kJ} \qquad (5.29) \end{aligned}$$

Eliminating ΔU from (5.28) and (5.29), we get

$$12.46\, n\, (T_f - T_o) = 250 \qquad (5.30)$$

Now, there are three independent equations (5.26), (5.27) and (5.30), and four unknowns P_f, T_o, T_f and n. We require one more independent equation to solve for the unknowns.

We have used all the three tools available, and there are no other means to get that missing equation from the data available in the problem statement. Therefore, we have to work with what is available to determine the value of P_f. There is a way to do so. Let us eliminate T_f and T_o from (5.30) using (5.26) and (5.27), respectively, as below.

$$12.46\, n \left[\frac{0.5\, P_f}{8.314\, n} - \frac{500 \times 0.5}{8.314\, n} \right] = 250$$

which gives P_f = 833.6 kPa.

Example 5.4

An unidentified gas of mass 6 g is contained in a piston-cylinder device of negligible heat capacity. The pressure of the gas is 1.7 bar, its volume 0.002 m^3, and its temperature 27°C. The gas expands receiving 445 J of heat. The temperature of the gas becomes 115°C, its pressure reduced, and its volume increased. The work done by the gas on its surroundings is 100 J. Determine the values of C_v, C_p, and the molar mass of the gas, assuming ideal gas behaviour.

Solution to Example 5.4

Since ideal gas behaviour is assumed, we use (5.8) to determine the value of C_v. Taking C_v as a constant, we get from (5.8)

$$\Delta u = C_v \left(T_f - T_o\right) \tag{5.31}$$

Since $T_o = 27°\mathrm{C}$ and $T_f = 115°\mathrm{C}$, we get $(T_f - T_o) = 88°\mathrm{C}$, which is equivalent to 88 K since a unit change in °C is the same as a unit change is K. Therefore, (5.31) gives

$$C_v = \Delta u/(88\ \mathrm{K}) \tag{5.32}$$

Now we need to calculate the internal energy change. The gas receives 445 J of heat and do 100 J of work. Taking the gas as the closed system, the first law of thermodynamics is applied as

$$\Delta U = Q_{in} + W_{in} = 445\ \mathrm{J} + (-100\ \mathrm{J}) = 345\ \mathrm{J}$$

The mass of the gas is 6 g, and thus

$$\Delta u = \Delta U/m = 345\ \mathrm{J}/6\ \mathrm{g} = 57.5\ \mathrm{J/g} \tag{5.33}$$

Eliminating Δu from (5.32) and (5.33), we get

$$C_v = \frac{57.5\ \mathrm{J/g}}{88\ \mathrm{K}} = 0.653\ \mathrm{J/g\cdot K} = 0.653\ \mathrm{kJ/kg\cdot K} \tag{5.34}$$

It is possible to determine the value of C_p using $C_p = C_v + R$ given by (5.14) for an ideal gas, provided we know the values of C_v and R. We, of course, know the value of C_v in kJ/kg · K from (5.34), and therefore we need the value of R in kJ/kg · K.

Since the values of pressure, volume, mass and temperature at the initial state of the gas is given in the problem, we find R using the ideal gas equation of state given by (5.3) as

$$R = \frac{(1.7 \times 100\ \mathrm{kPa})\ (0.002\ \mathrm{m}^3)}{(0.006\ \mathrm{kg})\ (273 + 27)\ \mathrm{K}} = 0.189\ \mathrm{kJ/kg\cdot K} \tag{5.35}$$

which is the specific gas constant.

Substituting the value of C_v from (5.34) and R from (5.35) in (5.14), we get the value of C_p as

$$\begin{aligned} C_p &= C_v + R \\ &= 0.653\ \mathrm{kJ/kg\cdot K} + 0.189\ \mathrm{kJ/kg\cdot K} = 0.842\ \mathrm{kJ/kg\cdot K} \end{aligned}$$

The molar mass could be found as

$$\begin{aligned} M &= \frac{\text{universal gas constant}}{\text{specific gas constant}} \\ &= \frac{8.314\ \mathsf{kJ/kmol \cdot K}}{0.189\ \mathsf{kJ/kg \cdot K}} = 43.99\ \frac{\mathsf{kg}}{\mathsf{kmol}} \approx 44\ \frac{\mathsf{kg}}{\mathsf{kmol}} \end{aligned}$$

Going through Table 5.2, we see that the numerical values of C_v, C_p and M evaluated above match with those of carbon dioxide.

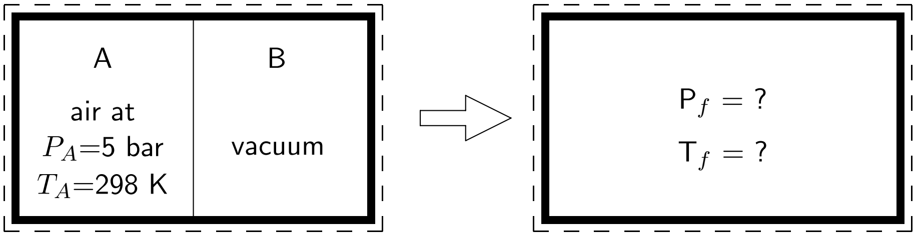
Example 5.5

A thermally insulated rigid box of negligible heat capacity contains two compartments of equal volume. Initially, one compartment contains air at 5 bar and 25°C, and the other is evacuated. The dividing partition of negligible mass, is then raptured. Calculate the final air temperature and pressure. It is a common assumption that air at low or moderate pressures behaves like an ideal gas. The value C_v for air at room temperature is 718 J/kg K.

Solution to Example 5.5

As shown in Figure 5.3, compartment A contains air initially, and compartment B contains nothing, that is vacuum. When the partition is raptured, air in compartment A rushes into compartment B. Because air rushes to occupy the entire box, the pressure and temperature of air occupying the entire box have highly nonuniform distributions. If we allow a considerable length of time to pass, then it is possible for air occupying the entire box to reach an equilibrium state, having a final temperature T_f and a final pressure P_f uniformly distributed everywhere within the box. We need to find the numerical values of T_f and P_f.

To be able to work out this problem, it is important to choose the system with care. Let us choose the closed system to include air in compartment A, compartment B, partition, and the rigid vessel. The boundary of the system chosen is represented by the dashed line in Figure 5.3.



initial state,
with partition
($V_A = V_B$)

final state,
without partition

Figure 5.3 The initial and final states of the system of Example 5.5.

Since no heat or no work crosses the boundary, the first law applied to the chosen closed system yields

$$\Delta U = 0 \tag{5.36}$$

which means there is no change in the total internal energy of the closed system. The total internal energy change of the chosen closed system is made up of the following components:

$$\Delta U = \Delta U_{air\ in\ A} + \Delta U_{vessel} + \Delta U_{partition}$$

where compartment B makes no contribution towards the total internal energy change of the system since it contains nothing. The vessel has negligible heat capacity and the partition has negligible mass, and therefore their contributions to the total internal energy change of the system are negligible. Thus, we get

$$\Delta U = \Delta U_{air\ in\ A} = 0 \tag{5.37}$$

which means the internal energy of air that was initially occupying compartment A remains a constant.

Since the mass of this air remains the same, we conclude that the specific internal energy of air that was occupying compartment A remains a constant, which means

$$\Delta u_{air\ in\ A} = 0 \tag{5.38}$$

Air is assumed to behave as an ideal gas, and C_v of air is taken a constant. Therefore, with the help of (5.8), we expand (5.38) to give

$$C_v \left[T_f - T_A \right] = 0 \tag{5.39}$$

where T_A is the temperature of air occupying compartment A at the initial state, and T_f is the temperature of air occupying the entire box at the final state.

Equation (5.39) gives

$$T_f = T_A = 298 \text{ K} = 25°\text{C} \tag{5.40}$$

That is to say the temperature of air has not changed even though the volume occupied by air has doubled. This should not come as a surprise since the specific internal energy of air, taken as an ideal gas, is a function of temperature alone. There was no change in the specific internal energy of air according to (5.38), and therefore there is no change in its temperature, either.

Now, we have to find the final pressure P_f. Note that we have already used two of the three tools mentioned at the beginning of this section. We are left with only one tool which is the ideal gas equation of state. Application of the ideal gas equation of state to air at its initial and final states gives the following respective equations:

$$P_A\,V_A = n\,R\,T_A \quad \text{and} \tag{5.41}$$

$$P_f\,(V_A + V_B) = n\,R\,T_f \tag{5.42}$$

where P_A, V_A and T_A are the respective pressure, volume and temperature of air at its initial state, and P_f, $(V_A + V_B)$ and T_f are the respective pressure, volume and temperature of air at its final state. The amount of air occupying compartment A at the initial state is the same as that occupying the entire box at the final state, and is denoted by n.

Eliminating the term $(n\,R)$ from (5.41) and (5.42), we get

$$\frac{P_f\,(V_A + V_B)}{T_f} = \frac{P_A\,V_A}{T_A} \tag{5.43}$$

Compartments A and B are of equal volume, and therefore $V_A = V_B$. We also know from (5.40) that $T_f = T_A$. Using the above facts and the information that the pressure of air in compartment A at the initial state, P_A, is 5 bar in (5.43), we get

$$P_f = 2.5 \text{ bar}$$

Example 5.6

A thermally insulated, rigid vessel with negligible heat capacity is divided into two compartments A and B by a non heat conducting partition of negligible mass such that $V_A = 4\,V_B$. Each contains nitrogen which behaves ideally. Compartment A is at 1 bar and 300 K, and compartment B is at 4 bar and 600 K. Determine the equilibrium temperature and the equilibrium pressure reached after the removal of the partition. The value C_v for nitrogen may be assumed a constant at 0.743 kJ/kg · K.

Solution to Example 5.6

This problem is similar to that of **Example 5.5**, except for the fact that compartment B is not vacuum but has nitrogen in it. The system boundary shown by the dashed line of Figure 5.4 clearly defines the system, which consists of nitrogen in both compartments A and B, the partition and the rigid vessel. Since neither heat nor work crosses the system boundary, the first law applied to closed systems yields

$$\Delta U = 0 \tag{5.44}$$

which means that there is no change in the total internal energy of the system.

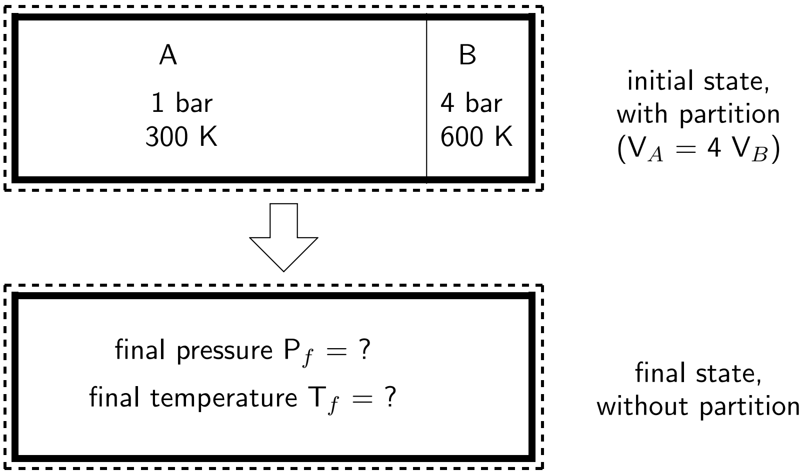


Figure 5.4 The initial and final states of the system of Example 5.6.

Ignoring the internal energy changes of the partition and the vessel, the total internal energy change of the system is written as

$$\Delta U = \Delta U\big|_{N_2\ in\ A} + \Delta U\big|_{N_2\ in\ B} = m_A\, \Delta u\big|_{N_2\ in\ A} + m_B\, \Delta u\big|_{N_2\ in\ B}$$

where m_A and m_B are the respective masses of nitrogen in compartments A and B at the initial state.

Since nitrogen contained in both the compartments A and B behaves as an ideal gas, and since nitrogen has constant C_v, we may expand the above expression as

$$\Delta U = m_A\, C_v\, (T_f - T_A) \; + \; m_B\, C_v\, (T_f - T_B) \tag{5.45}$$

where T_f is the temperature of nitrogen at the final state, T_A is the temperature of nitrogen in compartment A at the initial state, and T_B is the temperature of nitrogen in compartment B at the initial state.

Combining (5.44) and (5.45), we get

$$m_A\, C_v\, (T_f - T_A) \; + \; m_B\, C_v\, (T_f - T_B) \; = \; 0$$

Substituting the respective temperatures of nitrogen in compartments A and B at the initial state in the above equation and eliminating C_v, we get

$$m_A\, (T_f - 300\ \mathsf{K}) \; + \; m_B\, (T_f - 600\ \mathsf{K}) = 0 \tag{5.46}$$

The respective masses of nitrogen in A and in B at the initial state are evaluated using the ideal gas equation of the form $P\,V = m\,R\,T$ as

$$m_A = \frac{(1\ \mathsf{bar})\, V_A}{R\,(300\ \mathsf{K})} \quad \text{and} \quad m_B = \frac{(4\ \mathsf{bar})\, V_B}{R\,(600\ \mathsf{K})} \tag{5.47}$$

Substituting m_A and m_B of (5.47) in (5.46), we get

$$\frac{(1\ \mathsf{bar})\, V_A}{R\,(300\ \mathsf{K})}\, (T_f - 300\ \mathsf{K}) \; + \; \frac{(4\ \mathsf{bar})\, V_B}{R\,(600\ \mathsf{K})}\, (T_f - 600\ \mathsf{K}) = 0 \tag{5.48}$$

Since $V_A = 4\ V_B$, (5.48) gives $T_f = 400$ K.

Now, we have to determine P_f, the pressure at the final state. Note that we have already used all the three tools mentioned at the very beginning of this section. And, therefore, it may seem impossible to find the value of P_f. However, there is a fundamental law known as the principle of conservation of mass, which we have not used.

Application of the principle of conservation of mass to the given closed system yields

total mass of nitrogen in the vessel at the final state
= total mass of nitrogen in the vessel at the initial state

which gives

$$m_{final} = m_A + m_B \tag{5.49}$$

where m_{final} is the total mass of nitrogen in the vessel at the final state, and it is related to P_f by the ideal gas equation of state according to

$$m_{final} = \frac{P_f\,(V_A + V_B)}{R T_f} \tag{5.50}$$

Substituting m_A, m_B and m_{final} from (5.47) and (5.50), respectively, in (5.49), we get

$$\frac{P_f\,(V_A + V_B)}{R T_f} = \frac{(1\ \text{bar})\,V_A}{R\,(300\ \text{K})} + \frac{(4\ \text{bar})\,V_B}{R\,(600\ \text{K})} \tag{5.51}$$

Since $V_A = 4\ V_B$ and $T_f = 400$ K, (5.51) gives $P_f = 1.6$ bar.

Example 5.7

A thermally insulated rigid box of negligible heat capacity is divided into two compartments having a volume of 0.5 m^3 each by a piston that is kept from moving by a pin. The piston does not permit the gases to leak into each other. Initially, one compartment contains nitrogen at 5 bar and the other compartment contains hydrogen at 2 bar. The entire system is at 300 K. The mass of the piston is 2 kg and it is made up of copper of specific heat 0.386 kJ/kg$\cdot$°C. The pin holding the piston is removed and the piston is allowed to reach an equilibrium position. The piston is assumed to move without friction. Determine the pressures, temperatures and volumes of the gases at the final equilibrium state. Assume ideal gas behaviour. Take C_v as 0.743 kJ/kg$\cdot$K for nitrogen, and C_v as 10.199 kJ/kg$\cdot$K for hydrogen. Molar mass of nitrogen is 28 kg/kmol and that of hydrogen is 2 kg/kmol.

Solution to Example 5.7

The system boundary shown by the dashed line of Figure 5.5 clearly defines the system, which consists of nitrogen in compartment A, hydrogen in compartment B, the piston and the rigid vessel.

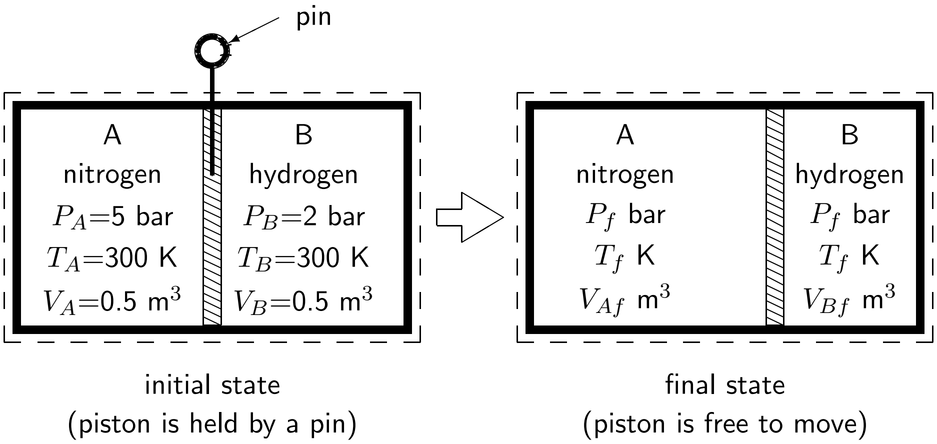


Figure 5.5 The initial and final states of the system of Example 5.7.

Once the pin holding the piston is removed, the piston is free to move until the net force acting on the piston becomes zero. The force balance on the piston in the absence of friction is written as follows:

pressure in compartment A x cross-sectional area of the piston =
pressure in compartment B x cross-sectional area of the piston,

which gives that, at the final equilibrium state, the pressures in the two compartment on either side of the piston equal each other. Let us denote this pressure as P_f (see Figure 5.5).

Since the piston is made up of copper, which is a good heat conductor, the temperatures of the gases as well as the piston are all the same; 300 K at the initial state and T_f at the final state (see Figure 5.5). The respective volumes of nitrogen and hydrogen at the final state are given by V_{Af} and V_{Bf} (see Figure 5.5). We are to determine the numerical values of P_f, T_f, V_{Af} and V_{Bf}.

Since neither heat nor work crosses the system boundary, the first law applied to closed systems yields $\Delta U = 0$, which means that there is no change in the total internal energy of the system. Ignoring the internal energy change of the vessel of negligible heat capacity, the total internal energy change of the system is written as

$$\Delta U = \Delta U_A + \Delta U_B + \Delta U_p = 0 \tag{5.52}$$

where the subscript A denotes the nitrogen in compartment A, B denotes the hydrogen in compartment B, and p denotes the piston.

Expanding (5.52) in terms of masses and specific heats, we get

$$m_A\,(C_v)_A\,(T_f - 300) + m_B\,(C_v)_B\,(T_f - 300) + m_p\,(\text{specific heat})_p\,(T_f - 300) = 0 \tag{5.53}$$

in which the initial temperatures of hydrogen, nitrogen and piston are all 300 K, and the final temperatures are all T_f K. When rearranged, (5.53) gives

$$[m_A\,(C_v)_A + m_B\,(C_v)_B + m_p\,(\text{specific heat})_p]\ [T_f - 300\ \text{K}] = 0 \tag{5.54}$$

Since $[m_A\,(C_v)_A + m_B\,(C_v)_B + m_p\,(\text{specific heat})_p]$ of (5.54) takes a finite value, we get

$$T_f = 300\ \text{K} \tag{5.55}$$

Now, we go on to determine P_f, which is the final pressure in both the compartments, using the ideal gas equation of state and the principle of conservation of mass as follows. The masses of gases in compartments A and B remain constant during the process. Therefore, applying the ideal gas equation of state to the gases in compartments A and B at the initial and final states yields the following equations, with reference to Figure 5.5:

$$\frac{P_f\,V_{Af}}{T_f} = \frac{(5\ \text{bar})\,(0.5\ \text{m}^3)}{300\ \text{K}} \tag{5.56}$$

and

$$\frac{P_f\,V_{Bf}}{T_f} = \frac{(2\ \text{bar})\,(0.5\ \text{m}^3)}{300\ \text{K}} \tag{5.57}$$

Substituting the value of T_f from (5.55) in (5.56) and (5.57), we get

$$P_f\,V_{Af} = (5\ \text{bar})\,(0.5\ \text{m}^3) \quad \text{and} \tag{5.58}$$

$$P_f\,V_{Bf} = (2\ \text{bar})\,(0.5\ \text{m}^3) \tag{5.59}$$

which has three unknowns P_f, V_{Af} and V_{Bf}.

There are no other equations except (5.58) and (5.59) that are in terms of the unknowns. We therefore need one more independent equation in terms of one or more of the three variables P_f, V_{Af} and V_{Bf}, to solve for the unknowns. We have indeed used up all three of our tools along with the principle of conservation of mass. The situation seems hopeless except for the fact that the total volume of the system remains a constant. Therefore, we get

$$V_{Af} + V_{Bf} = 1 \text{ m}^3 \tag{5.60}$$

Substituting V_{Af} from (5.58) and V_{Bf} from (5.59) in (5.60), we get

$$\frac{(5 \text{ bar})\,(0.5 \text{ m}^3)}{P_f} + \frac{(2 \text{ bar})\,(0.5 \text{ m}^3)}{P_f} = 1 \text{ m}^3 \tag{5.61}$$

and thus

$$P_f = 3.5 \text{ bar} \tag{5.62}$$

Substituting the value of P_f from (5.62) in (5.58), we get

$$V_{Af} = \frac{(5 \text{ bar})\,(0.5 \text{ m}^3)}{3.5 \text{ bar}} = 0.714 \text{ m}^3 \tag{5.63}$$

Substituting the value of V_{Af} from (5.63) in (5.60), we get $V_{Bf} = 0.286 \text{ m}^3$.

In working out thermodynamic problems on closed simple compressible systems with ideal gases, we started out with the following three tools:

Tool 1: The first law of thermodynamics applied to closed simple compressible systems, given by $Q_{in} + W_{in} = \Delta U$, which is also the principle of conservation of energy.

Tool 2: Evaluation of ΔU using data on specific heats, or data on internal energy itself.

Tool 3: The ideal gas equation of state that interrelates pressure, temperature, volume and mass (or amount of matter) of an ideal gas.

However, while solving the problems we realized that the following two tools must be added to the list:

Tool 4: Principle of conservation of mass.

Tool 5: Any other constraint stated in the problem, such as "the total volume remains constant".

Example 5.8

Show that we require more information about the problem to solve for the unknowns of **Example 5.7** if the heat conducting copper piston is replaced by a piston that is a perfect heat insulator.

Solution to Example 5.8

The problem of this example differs from that of **Example 5.7** in the aspect that the piston of this example does not allow heat to pass from one compartment to the other. Therefore, the final temperatures in the two compartments cannot be considered the same, as was in the **Solution to Example 5.7**.

The unknowns of the problem are P_f, which denotes the equal final pressures of gases in both the compartments, T_{Af} and T_{Bf}, which are the unequal final temperatures of gases in both the compartments, and V_{Af} and V_{Bf}, which are the unequal final volumes of the gases in both the compartments. To solve for the five unknowns, P_f, T_{Af}, T_{Bf}, V_{Af} and V_{Bf}, we require five independent equations in terms of one or more of the unknowns alone. Let us see how to get the equations required.

Using **Tool 1** and **Tool 2** we get

$$m_A\,(C_v)_A\,(T_{Af} - T_{Ao}) + m_B\,(C_v)_B\,(T_{Bf} - T_{Bo}) = 0 \tag{5.64}$$

where m_A, $(C_v)_A$, T_{Ao} and T_{Af} are the respective mass, C_v, initial temperature and final temperature of nitrogen in one compartment, and m_B, $(C_v)_B$, T_{Bo} and T_{Bf} are the respective mass, C_v, initial temperature and final temperature of hydrogen in the other compartment.

Substituting whatever is known from the problem statement in (5.64), we get

$$m_A\,(0.743\ \text{kJ/kg}\cdot\text{K})\,(T_{Af} - 300\ \text{K}) + m_B\,(10.199\ \text{kJ/kg}\cdot\text{K})\,(T_{Bf} - 300\ \text{K}) = 0 \tag{5.65}$$

The masses of the gases are found using **Tool 3** as follows:

$$m_A = \frac{(5\ \text{bar} \times\ 100\ \text{kPa/bar})\,(0.5\ \text{m}^3)}{(8.314/28\ \text{kJ/kg}\cdot\text{K})\,(300\ \text{K})} = 2.81\ \text{kg} \tag{5.66}$$

$$m_B = \frac{(2\ \text{bar} \times\ 100\ \text{kPa/bar})\,(0.5\ \text{m}^3)}{(8.314/2\ \text{kJ/kg}\cdot\text{K})\,(300\ \text{K})} = 0.08\ \text{kg} \tag{5.67}$$

Substituting m_A of (5.66) and m_B of (5.67) in (5.65), we get

$$2.81 \times 0.743 \times (T_{Af} - 300) + 0.08 \times 10.199 \times (T_{Bf} - 300) = 0 \tag{5.68}$$

which is one of the five equations in terms of the unknowns alone.

Since the masses of gases in compartments A and B stay the same during the process, use of **Tool 4** gives

$$m_{Af} = m_{Ao} \quad \text{and} \quad m_{Bf} = m_{Bo} \tag{5.69}$$

Using **Tool 3**, (5.69) can be expanded to

$$\frac{P_f\, V_{Af}}{T_{Af}} = \frac{(500\ \text{kPa})\,(0.5\ \text{m}^3)}{300\ \text{K}} \quad \text{and} \tag{5.70}$$

$$\frac{P_f\, V_{Bf}}{T_{Bf}} = \frac{(200\ \text{kPa})\,(0.5\ \text{m}^3)}{300\ \text{K}} \tag{5.71}$$

which are two more independent equations in terms of the unknowns alone.

Tool 5 gives

$$V_{Af} + V_{Bf} = 1\ \text{m}^3 \tag{5.72}$$

which is yet another independent equation in terms of the unknowns alone.

We now have four independent equations (5.68), (5.70), (5.71) and (5.72) to solve for the five unknowns P_f, T_{Af}, T_{Bf}, V_{Af} and V_{Bf}. From the problem statement, it is impossible to get one more independent equation in terms of one or more of the unknowns. Therefore, more information about the problem should be provided in order to solve for the unknowns, if the heat conducting copper piston of **Example 5.7** is replaced by a piston that is a perfect heat insulator.

5.10 Summary

- The ideal gas equation of state is given by

  $$PV = nRT \tag{5.1}$$

  or by

  $$PV = mRT \tag{5.3}$$

  where P is the absolute pressure in kPa, V is the total volume in m^3, and T is the absolute temperature in K. In (5.1), n is the amount of gas in kmol, and therefore R is the universal gas constant which takes the value 8.314 kJ/kmol·K for all gases. In (5.3), m is the mass of gas in kg, and therefore R is the specific gas constant in kJ/kg·K, the numerical value of which depends on the gas concerned.

- Specific gas constant is related to the universal gas constant by

$$\text{specific gas constant} = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{\text{M (in kg/kmol)}}$$

where $M = m/n$, is the molar mass.

- The ideal gas equation of state is also given by

$$Pv = RT \tag{5.4}$$

where P is the absolute pressure in kPa and T is the absolute temperature in K. If v is the molar volume in m^3/kmol, then R is the universal gas constant. If v is the specific volume in m^3/kg, then R is the specific gas constant.

- For an ideal gas, properties u, h, C_v and C_p depend only on temperature.
- For an ideal gas, specific (or molar) enthalpy is given by

$$h = u + RT \tag{5.10}$$

- Specific heat at constant volume for an ideal gas is defined as

$$C_v = \frac{du}{dT} \tag{5.6}$$

- Specific (or molar) internal energy change of an ideal gas undergoing any process can be evaluated using

$$\Delta u = \int_{T_o}^{T_f} C_v \, dT \tag{5.8}$$

Note that (5.8) could also be used to evaluate Δu for substances other than ideal gas provided the process is executed at constant volume.

- To evaluate Δu using (5.8), C_v must be known either as a function of T or be approximated to a constant. It is also common to obtain Δu as (u_f - u_o), where u_f and u_o are obtained from ideal-gas (or zero-pressure) property tables.

- Specific heat at constant pressure for an ideal gas is defined as

$$C_p = \frac{dh}{dT} \tag{5.11}$$

- Specific (or molar) enthalpy change of an ideal gas undergoing any process can be evaluated using

$$\Delta h = \int_{T_o}^{T_f} C_p \, dT \tag{5.13}$$

  Note that (5.13) could also be used to evaluate Δh for substances other than ideal gas provided the process is executed at constant pressure.

- To evaluate Δh using (5.13), C_p must be known either as a function of T or be approximated to a constant. It is also common to obtain Δh as (h_f - h_o), where h_f and h_o are obtained from ideal-gas (or zero-pressure) property tables.

- Specific heats of an ideal gas is related to each other by

$$C_p = C_v + R \tag{5.14}$$

- Specific heat ratio (or the isentropic exponent) is defined as

$$\gamma = C_p/C_v \tag{5.15}$$

- Specific heats of an ideal gas are given in terms of γ and R as follows:

$$C_v = \frac{R}{\gamma - 1} \tag{5.16}$$

$$C_p = \frac{\gamma R}{\gamma - 1} \tag{5.17}$$

- In working out thermodynamic problems on closed simple compressible systems with ideal gases, the following tools are used:

  Tool 1: Principle of conservation of mass.

  Tool 2: The first law of thermodynamics applied to closed simple compressible systems, given by $Q_{in} + W_{in} = \Delta U$, which is also the principle of conservation of energy.

  Tool 3: The ideal gas equation of state that interrelates pressure, temperature, volume and mass (or amount of matter) of an ideal gas.

  Tool 4: Evaluation of ΔU using data on specific heats, or data on internal energy itself.

  Tool 5: Any other constraint stated in the problem.